Constructing generic non-extremal black-hole solutions of N=2, d=4,5 Supergravity

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Work done in collaboration with

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Plan of the Talk:

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In this talk I will present a general ansatz and a general formalism to construct non-extremal black-hole and black-brane solutions and we will study some examples. First, we will review some general results.

We are interested in explicit solutions of non-extremal black holes and black branes. We are going to use a generalization of the *FGK formalism (Ferrara-Gibbons-Kallosh, 1997)* which has been used mainly to study extremal black-hole solutions in only 4 dimensions.

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We start by reviewing the FGK formalism

for black holes and black branes.

2 – FGK formalism for black p-branes in d dimensions

Consider the generic d-dimensional spacetime action describing scalars ϕ^i and $(p+1)$-form potentials $A^{\Lambda}_{(p+1)}$ coupled to gravity:

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\begin{aligned}
I \;=\; & \int d^dx\sqrt{|g|}\,\Big\{R+\mathcal{G}_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j \\
& +4\tfrac{(-1)^p}{(p+2)!}\Big[I_{\Lambda\Sigma}(\phi)F^{\Lambda}_{(p+2)}\cdot F^{\Sigma}_{(p+2)}+\xi^2 R_{\Lambda\Sigma}(\phi)F^{\Lambda}_{(p+2)}\star F^{\Sigma}_{(p+2)}\Big]\Big\}\,,
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where the last term occurs only when $p=\tilde{p}=(d-4)/2$ and

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We want to find solutions describing single, static, charged, regular, black p-branes with flat worldvolume in the directions $\vec{y}_{(p)}=(y_1,\cdots,y_p)$ living in a spacetime of $d=p+\tilde{p}+4$ dimensions.

Our general ansatz for the metric only contains an independent function $\tilde{U}(\rho)$.

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\begin{aligned}
ds^2_{(d)} &= e^{\frac{2}{p+1}\tilde{U}}\left[e^{\frac{2p}{p+1}r_0\rho}dt^2 - e^{-\frac{2}{p+1}r_0\rho}d\vec{y}^{\,2}_{(p)}\right] - e^{-\frac{2}{\tilde{p}+1}\tilde{U}}\gamma_{(\tilde{p}+3)\,mn}dx^m dx^n\,, \\
\gamma_{(\tilde{p}+3)\,mn}dx^m dx^n &\equiv \left[\frac{r_0}{\sinh(r_0\rho)}\right]^{\frac{2}{\tilde{p}+1}}\left[\left(\frac{r_0}{\sinh(r_0\rho)}\right)^2\frac{d\rho^2}{(\tilde{p}+1)^2} + d\Omega^2_{(\tilde{p}+2)}\right],
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and T is the *Hawking temperature*

$$(2r_0)^{\frac{1}{p+1}} = \frac{4\pi}{\tilde{p}+1} T \tilde{S}^{\frac{(d-2)}{(p+1)(\tilde{p}+2)}} \, .$$

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This relation is true with the same r_0 for both inner and outer horizons.

With this formalism we will be able to compute the entropies of the inner ($-$) and outer ($+$) horizons and check that the product

$$\tilde{S}_+ \tilde{S}_-$$

is a moduli-independent combination of conserved quantities.

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What is r_0 in more general cases?

The effective action for $\tilde{U}(\rho), \phi^i(\rho)$ is

$$I_{\rm eff}[\tilde{U}, \phi^i] = \int d\tau \left\{ (\dot{\tilde{U}})^2 + \tfrac{(p+1)(\tilde{p}+2)}{d-2} \mathcal{G}_{ij} \dot{\phi}^i \dot{\phi}^j - e^{2\tilde{U}} V_{\rm BB} + {r_0}^2 \right\} ,$$

where we have defined the black-brane potential

$$-V_{\rm BB}(\phi, \mathcal{Q}) \equiv -\tfrac{1}{2} \mathcal{Q}^M \mathcal{Q}^N \mathcal{M}_{MN}(\phi) ,$$

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$$(\mathcal{Q}^M) = \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} \qquad (\mathcal{M}_{MN}) \equiv \begin{pmatrix} (I - \xi^2 R I^{-1} R)_{\Lambda\Sigma} & \xi^2 (R I^{-1})_\Lambda{}^\Sigma \\ & \\ -(I^{-1}R)^\Lambda{}_\Sigma & (I^{-1})^{\Lambda\Sigma} \end{pmatrix} ,$$

are $\mathrm{O}(n, n)$ (resp. $\mathrm{Sp}(n, n)$) vector and matrix when $\xi^2 = +1$ (resp. -1). (In general $R_{\Lambda\Sigma} = p^\Lambda = 0$ and the duality group is just $\mathrm{SO}(n)$).

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Finding a p-black brane in d dimensions with charges p, q is equivalent to solving the above mechanical system for $\tilde{U}(\rho), \phi^i(\rho)$.

We can now use the equations of motion to derive general results for black branes, generalizing those obtained by FGK for 4-dimensional black holes.

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☞ The near-horizon geometry is always $AdS_{p+2} \times S^{\tilde{p}+2}$ with the AdS_{p+2} and $S^{\tilde{p}+2}$ radii both equal to $\tilde{S}^{1/2}$.

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{r_0}^2 = \frac{[(p+1)(\tilde{p}+2)T_p + p(\tilde{p}+1)r_0]^2}{(d-2)^2} + \frac{(p+1)(\tilde{p}+2)}{(d-2)}\mathcal{G}_{ij}(\phi_\infty)\Sigma^i\Sigma^j + V_{\rm bh}(\phi_\infty, q, p)\,,
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According to the no-hair "theorem" only $\Sigma^i = \Sigma^i(T_p, \phi^i_\infty, q, p)$ (*secondary hair*) are allowed for regular black branes.

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$$r_0{}^2 = \frac{[(p+1)(\tilde{p}+2)T_p + p(\tilde{p}+1)r_0]^2}{(d-2)^2} + \frac{(p+1)(\tilde{p}+2)}{(d-2)}\mathcal{G}_{ij}(\phi_\infty)\Sigma^i\Sigma^j + V_{\rm bh}(\phi_\infty, q, p)\,,$$

However, this expression is **useless**!

According to the no-hair "theorem" only $\Sigma^i = \Sigma^i(T_p, \phi^i_\infty, q, p)$ (*secondary hair*) are allowed for regular black branes.

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In the non-extremal case we need the complete <u>explicit</u> solution.

3 – Construction of explicit solutions: extremal supersymmetric

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We are going to review the black holes of (ungauged) $N = 2 \;\; d = 4$ Supergravity coupled to vector multiplets.

In order to find static extremal black holes one could try to integrate directly the equations of motion of the FGK formalism for the black-hole potential of $N = 2$ $d = 4$ theories:

$$-V_{\rm bh} = |\mathcal{Z}|^2 + \mathcal{G}^{ij^*}\mathcal{D}_i\mathcal{Z}\mathcal{D}_{j^*}\mathcal{Z}^*\,,$$

where $\mathcal{Z}$ is the central charge of the theory

$$\mathcal{Z}(\phi,p,q) \equiv \langle\, \mathcal{V}(\phi) \mid \mathcal{Q}\,\rangle \equiv \langle \begin{pmatrix} \mathcal{L}^\Lambda \\ \mathcal{M}_\Lambda \end{pmatrix} \mid \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix} \rangle \equiv p^\Lambda \mathcal{M}_\Lambda(\phi) - q_\Lambda \mathcal{L}^\Lambda(\phi)\,.$$

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There is a recipe to construct all the BPS ones.

(Behrndt, Lüst & Sabra (1997), Denef (2000), Lopes Cardoso, de Wit, Kappeli & Mohaupt, Meessen, O. (2006))

1. For some complex X, define the Kähler-neutral, real, symplectic vectors $\mathcal{R}$ and $\mathcal{I}$

$$\mathcal{R} + i\mathcal{I} \equiv \mathcal{V}/X \, .$$

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2. The components of $\mathcal{I}$ are given by a symplectic vector real functions harmonic in the 3-dimensional transverse space. For single black holes ($\tau \equiv -\rho$):

$$\begin{pmatrix} \mathcal{I}^{\Lambda} \\ \\ \mathcal{I}_{\Lambda} \end{pmatrix} = \begin{pmatrix} H^{\Lambda}(\tau) \\ \\ H_{\Lambda}(\tau) \end{pmatrix} = \begin{pmatrix} H^{\Lambda}{}_{\infty} - \frac{1}{\sqrt{2}} p^{\Lambda} \tau \\ \\ H_{\Lambda\,\infty} - \frac{1}{\sqrt{2}} q_{\Lambda} \tau \end{pmatrix} ,$$

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5. The function $U(\tau)$ of the FGK formalism is given by

$$e^{-2U} = \langle\, \mathcal{R} \mid \mathcal{I}\,\rangle = \mathcal{I}^{\Lambda}\mathcal{R}_{\Lambda} - \mathcal{I}_{\Lambda}\mathcal{R}^{\Lambda}\,.$$

The asymptotic values of the harmonic functions, H^M_∞ satisfying the condition $N = \langle\, H_\infty \mid \mathcal{Q}\,\rangle = 0$ have the general form

$$H^M{}_\infty = \pm\sqrt{2}\,\Im\mathrm{m}\left(\mathcal{V}^M_\infty \frac{\mathcal{Z}^*_\infty}{|\mathcal{Z}_\infty|}\right)\,, \qquad \mathcal{Z}_\infty \equiv \mathcal{Z}(\phi_\infty, p, q)\,, \qquad \mathcal{V}^M_\infty \equiv \mathcal{V}^M(\phi_\infty)\,.$$

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One can check in the explicit solutions all the properties predicted by the FGK formalism.

In this case the complete explicit solutions do not give much more information than the attractors, but they are going to be used as starting point for the construction of non-extremal solutions.

4 – Construction of explicit solutions: non-extremal

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The following prescription to deform the extremal supersymmetric solutions of $N = 2$ $d = 4$ Supergravity theories has been given in Galli, O., Perz & Shahbazi (2011):

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Then, the non-extremal solution is given by

$$U(\tau) = U_{\mathrm{e}}[H(\tau)] + r_0\tau\,, \qquad Z^i(\tau) = Z^i{}_{\mathrm{e}}[H(\tau)]\,,$$

where now the functions H are assumed to be of the form

$$H^M = a^M + b^M\, e^{2r_0\tau}\,,$$

and the constants a^M, b^M have to be determined by explicitly solving the e.o.m.

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It has been shown that it is possible to rewrite the FGK effective action using the $H^M(\tau)$ as variables that replace $U(\tau)$ and $\phi^i(\tau)$ (Mohaupt & Waite `arXiv:0906.3451`, Mohaupt & Vaughan `arXiv:1006.3439` & `arXiv:1112.2876`, Meessen, O., Perz & Shahbazi `arXiv:1112.3332`). This confirms our hypothesis.

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More on this, later.

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5 – A complete example: $\overline{\mathbb{CP}}^n$ model

This model has n scalars Z^i that parametrize the coset space $SU(1,n)/SU(n)$. We add for convenience $Z^0 \equiv 1$, so we have

$$(Z^\Lambda) \equiv (1, Z^i)\,, \qquad (Z_\Lambda) \equiv (1, Z_i) = (1, -Z^i)\,, \qquad (\eta_{\Lambda\Sigma}) = \mathrm{diag}(+ - \cdots -)\,.$$

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The covariantly holomorphic symplectic section reads $\mathcal{V} = e^{\mathcal{K}/2}\begin{pmatrix} Z^\Lambda \\ \\ -\frac{i}{2} Z_\Lambda \end{pmatrix}$.

It is convenient to define the complex charge combinations $\Gamma_\Lambda \equiv q_\Lambda + \frac{i}{2}\eta_{\Lambda\Sigma} p^\Sigma$.

In this model the central charge $\mathcal{Z}$, its holomorphic Kähler -covariant derivative and the black-hole potential are

$$\begin{aligned}
\mathcal{Z} &= e^{\mathcal{K}/2} Z^{\Lambda}\Gamma_{\Lambda}\,, \\
\mathcal{D}_i\mathcal{Z} &= e^{3\mathcal{K}/2} Z^{*}_{i} Z^{\Lambda}\Gamma_{\Lambda} - e^{\mathcal{K}/2}\Gamma_{i}\,, \\
|\tilde{\mathcal{Z}}|^2 &\equiv \mathcal{G}^{ij^*}\mathcal{D}_i\mathcal{Z}\mathcal{D}_{j^*}\mathcal{Z}^* = e^{\mathcal{K}}|Z^{\Lambda}\Gamma_{\Lambda}|^2 - \Gamma^{*\,\Lambda}\Gamma_{\Lambda}\,, \\
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In $N=2$ theories, in the extremal case $|\mathcal{Z}|$ plays the rôle of superpotential W. $|\tilde{\mathcal{Z}}|$ plays here the rôle of “fake” superpotential.

The **extremal** case

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We start by calculating the critical points of the black-hole potential:

$$\mathcal{G}^{ij^*}\partial_{j^*}V_{\rm bh} = 2\,Z^{\Lambda}\Gamma_{\Lambda}\left(\Gamma^{*\,i} - \Gamma^{*\,0}Z^{i}\right) = 0 \quad \Rightarrow \left\{ \begin{array}{l} Z^{i}{}_{\rm h} = \Gamma^{*\,i}/\Gamma^{*\,0}\,, \\ (\text{isolated, supersymmetric attractor}) \\ Z^{\Lambda}{}_{\rm h}\Gamma_{\Lambda} = 0\,, \\ (\text{hypersurface of non}-\text{supersymmetric} \\ attractors) \end{array} \right.$$

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Attractor	$e^{-\mathcal{K}_{\rm h}}$	$\lvert\mathcal{Z}_{\rm h}\rvert^2$	$\lvert\tilde{\mathcal{Z}}_{\rm h}\rvert^2$	$-V_{\rm bhh}$	M
$Z^{i\,\rm susy}_{\rm h} = \Gamma^{*\,i}/\Gamma^{*\,0}$	$\Gamma^{*\,\Lambda}\Gamma_{\Lambda} > 0$	$\Gamma^{*\,\Lambda}\Gamma_{\Lambda}$	0	$\Gamma^{*\,\Lambda}\Gamma_{\Lambda}$	$\lvert\mathcal{Z}_{\infty}\rvert$
$Z^{\Lambda\,\rm nsusy}_{\rm h}\Gamma_{\Lambda} = 0$	$-\Gamma^{*\,\Lambda}\Gamma_{\Lambda} > 0$	0	$-\Gamma^{*\,\Lambda}\Gamma_{\Lambda}$	$-\Gamma^{*\,\Lambda}\Gamma_{\Lambda}$	$\lvert\tilde{\mathcal{Z}}_{\infty}\rvert$

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Defining, for convenience

$$\mathcal{H}_\Lambda \equiv H_\Lambda + \tfrac{i}{2}\eta_{\Lambda\Sigma}H^\Sigma \equiv e^{\mathcal{K}_\infty/2}\frac{\mathcal{Z}_\infty}{|\mathcal{Z}_\infty|}Z^*_{\Lambda\,\infty} - \tfrac{1}{\sqrt{2}}\Gamma_\Lambda\tau$$

the metric function and the scalars are

$$e^{-2U} = 2\mathcal{H}^{*\,\Lambda}\mathcal{H}_\Lambda\,, \qquad\qquad Z^i = \frac{\mathcal{R}^i + i\mathcal{I}^i}{\mathcal{R}^0 + i\mathcal{I}^0} = \frac{\mathcal{H}^{*i}}{\mathcal{H}^{*0}}\,.$$

Non-extremal solutions

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Our Ansatz for the non-extremal solution is

$$e^{-2U} = e^{-2[U_{\rm e}(\mathcal{H})+r_0\tau]}\,, \qquad e^{-2U_{\rm e}(\mathcal{H})} = 2\mathcal{H}^{*\,\Lambda}\mathcal{H}_{\Lambda}\,, \qquad Z^i = Z^i{}_{\rm e}(\mathcal{H}) = \mathcal{H}^{*\,i}/\mathcal{H}^{*\,0}\,,$$

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where $\mathcal{H}^\Lambda \equiv A^\Lambda + B^\Lambda e^{2r_0\tau}\,, \quad \Lambda = 0, \cdots, n.$

The $2(n+1)$ complex constants A_Λ, B_Λ are found by imposing the e.o.m. $(f \equiv e^{r_0\tau})$

$$\begin{aligned}
\ddot{U}_{\rm e} - (\dot{U}_{\rm e})^2 - \mathcal{G}_{ij^*}\dot{Z}^i\dot{Z}^{*\,j^*} &= 0\,,\\
(2r_0)^2\left[f\ddot{U}_{\rm e} + \dot{U}_{\rm e}\right] + e^{2U_{\rm e}}V_{\rm bh} &= 0\,,\\
(2r_0)^2\left[f\left(\ddot{Z}^i + \mathcal{G}^{ij^*}\partial_k\mathcal{G}_{lj^*}\dot{Z}^k\dot{Z}^l\right) + \dot{Z}^i\right] + e^{2U_{\rm e}}\mathcal{G}^{ij^*}\partial_{j^*}V_{\rm bh} &= 0\,.
\end{aligned}$$

The e.o.m. are solved if the the constants satisfy the **algebraic** equations

$$
\begin{aligned}
\Im\mathrm{m}(B^{*\,\Lambda}A_\Lambda) &= 0\,,\\
A^{*\,\Lambda}A^\Sigma\xi_{\Lambda\Sigma} &= 0\,,\\
(A^{*\,\Lambda}B^\Sigma + B^{*\,\Lambda}A^\Sigma)\xi_{\Lambda\Sigma} &= 0\,,\\
B^{*\,\Lambda}B^\Sigma\xi_{\Lambda\Sigma} &= 0\,,\\
(2r_0)^2(B_i^*A_0^* - B_0^*A_i^*)A^{*\,\Lambda}A_\Lambda + (\Gamma_i^*A_0^* - \Gamma_0^*A_i^*)A^{*\,\Lambda}\Gamma_\Lambda &= 0\,,\\
-(2r_0)^2(B_i^*A_0^* - B_0^*A_i^*)B^{*\,\Lambda}B_\Lambda + (\Gamma_i^*B_0^* - \Gamma_0^*B_i^*)B^{*\,\Lambda}\Gamma_\Lambda &= 0\,,\\
(\Gamma_i^*A_0^* - \Gamma_0^*A_i^*)A^{*\,\Lambda}\Gamma_\Lambda + (\Gamma_i^*B_0^* - \Gamma_0^*B_i^*)B^{*\,\Lambda}\Gamma_\Lambda &= 0\,,
\end{aligned}
$$

where $\xi_{\Lambda\Sigma} \equiv 2\left(\Gamma_\Lambda\Gamma_\Sigma^* + 8{r_0}^2A_\Lambda B_\Sigma^*\right) - \eta_{\Lambda\Sigma}\left(\Gamma^\Omega\Gamma_\Omega^* + 8{r_0}^2A^\Omega B_\Omega^*\right)\,.$

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B^{*\,\Lambda}B^{\Sigma}\xi_{\Lambda\Sigma} &= 0\,,\\
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No differential equations remain to be solved!

Furthermore, we need to normalize the metric at spatial infinity and relate A_Λ, B_Λ to the physical parameters:

$$
\begin{aligned}
2(A^{*\,\Lambda} + B^{*\,\Lambda})(A_\Lambda + B_\Lambda) &= 1\,, \\
4\Re\mathrm{e}[B^{*\,\Lambda}(A_\Lambda + B_\Lambda)] &= 1 - M/r_0\,, \\
\frac{A^{*\,i} + B^{*\,i}}{A^{*\,0} + B^{*\,0}} &= Z^i{}_\infty\,.
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The solution can be found and it is

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\begin{aligned}
A_\Lambda &= \pm\frac{e^{\mathcal{K}_\infty/2}}{2\sqrt{2}}\left\{Z^*_{\Lambda\,\infty}\left[1 + \frac{(M^2 - e^{\mathcal{K}_\infty}|Z^{*\,\Sigma}_\infty\Gamma^*_\Sigma|^2)}{Mr_0}\right] + \frac{\Gamma_\Lambda Z^{*\,\Sigma}{}_\infty\Gamma^*_\Sigma}{Mr_0}\right\}, \\
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$$

Here $M^2{r_0}^2 = (M^2 - |\mathcal{Z}_\infty|^2)(M^2 - |\tilde{\mathcal{Z}}_\infty|^2)$, and one can show that the metric is regular in all the ${r_0}^2 > 0$ cases.

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   We get *harmonic functions with different coefficients*:

$$\mathcal{H}_\Lambda \stackrel{M \to |\tilde{\mathcal{Z}}_\infty|}{\longrightarrow} \pm \frac{e^{\mathcal{K}_\infty/2}}{2\sqrt{2}} \left\{ Z^*_{\Lambda\,\infty} - \frac{1}{|\tilde{\mathcal{Z}}_\infty|} \left[-Z^*_{\Lambda\,\infty} \Gamma^{*\,\Sigma}\Gamma_\Sigma + \Gamma_\Lambda Z^{*\,\Sigma}_\infty \Gamma^*_\Sigma \right] \tau \right\} .$$

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   On the event horizon $\tau \to -\infty$ the scalars $Z^i = \mathcal{H}^{*\,i}/\mathcal{H}^{*\,0}$ take the values

$$Z^{*\,i}_{\rm h} = \frac{\Gamma^i Z^{*\,\Lambda}_\infty \Gamma^*_\Lambda - Z^{*\,i}_\infty \Gamma^{*\,\Sigma}\Gamma_\Sigma}{\Gamma^0 Z^{*\,\Gamma}_\infty \Gamma^*_\Gamma - \Gamma^{*\,\Omega}\Gamma_\Omega} ,$$

   which *depend manifestly on the asymptotic values*.

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Since $M^2 r_0{}^2 = (M^2 - |\mathcal{Z}_\infty|^2)(M^2 - |\tilde{\mathcal{Z}}_\infty|^2)$ there are two $r_0 \to 0$ (extremal) limits:

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which *depend manifestly on the asymptotic values*.

There is no attractor behavior in a proper sense.

The structure of the extremal non-supersymmetric solution as function of the H^Ms is the same as in the supersymmetric case.

However, no simple *substitution recipe* could have led to it.

Physical properties of the non-extremal solutions

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One can compute the "entropies" of the inner and outer horizons (event horizon (+) and Cauchy horizon) at $\tau \to -\infty$ and $\tau \to +\infty$ resp.:

$$S_\pm/\pi = (M^2 - |\mathcal{Z}_\infty|^2) \pm (M^2 - |\tilde{\mathcal{Z}}_\infty|^2) \pm 2Mr_0\,.$$

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$$S_\pm/\pi = \sqrt{N_\mathrm{R}} \pm \sqrt{N_\mathrm{L}}\,, \quad \Rightarrow \quad S_+S_-/\pi^2 = N_\mathrm{R} - N_\mathrm{L} \in \mathbb{Z}\,.$$

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But, even though it is suggestive, *it is not unique*. We can also write

$$S_{\pm}/\pi = \left(\sqrt{N_{\rm R}} \pm \sqrt{N_{\rm L}}\right)^2\,,$$

with

$$N_{\rm R} \equiv M^2 - |\mathcal{Z}_\infty|^2\,, \qquad N_{\rm L} \equiv M^2 - |\tilde{\mathcal{Z}}_\infty|^2\,,$$

so

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There is an attractor behavior in the evaporation process.

6 – H-FGK formalism for $N = 2$, $d = 4$ supergravity

Or: Where the H^Ms come from

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In the $N = 2$ $d = 4$ case, the FGK formalism can be rewritten in different variables (Mohaupt & Vaughan `arXiv:1112.2876`, Meessen, O., Perz & Shahbazi `arXiv:1112.3332`)

$$U(\tau), Z^i(\tau) \ (2n_V + 1) \longrightarrow \begin{pmatrix} H^\Lambda \\ H_\Lambda \end{pmatrix} \equiv H^M \, , \ (2n_V + 2)$$

plus one constraint that will appear automatically.

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These variables transform linearly under the duality group, that acts on it as a subgroup of $\mathrm{Sp}(2n+2,\mathbb{R})$.

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We introduce an auxiliary function X and proceed as in the BPS case defining the Kähler-neutral, real, symplectic vectors $\mathcal{R}^M$ and $\mathcal{I}^M$

$$\mathcal{R}^M + i\mathcal{I}^M \equiv \mathcal{V}^{\mathcal{M}}/X\,.$$

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We know that the $\mathcal{R}^M$ can be expressed as a function of the $\mathcal{I}^M$s and vice-versa solving the *stabilization equations*. Then, we introduce *two dual sets of variables*

$$\tilde{H}_M \equiv \mathcal{R}_M \, , \qquad\qquad H^M \equiv \mathcal{I}^M \, .$$

We define the Hessian potential $\mathsf{W}(H) \equiv \tilde{H}_M(H) H^M$, or $\mathsf{W}(H) \equiv \tilde{H}_M H^M(H)$.

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Then, the FGK effective action can be written in the form

$$\begin{aligned} I_{\text{eff}}[H] \;\; = \;\; & \int d\tau \left\{ \tfrac{1}{2}\partial_M \partial_N \log \mathsf{W} \left(\dot{H}^M \dot{H}^N + \tfrac{1}{2}\mathcal{Q}^M \mathcal{Q}^N \right) \right. \\ & \left. \qquad + \left(\mathsf{W}^{-1} \dot{H}^M H_M \right)^2 + \left(\mathsf{W}^{-1} \mathcal{Q}^M H_M \right)^2 \right\}\,, \end{aligned}$$

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All the information about the model is encoded in the Hessian potential $\mathsf{W}(H)$.

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$$\partial_M \mathsf{W} \equiv \frac{\partial \mathsf{W}}{\partial H^M} = 2\tilde{H}_M \,, \qquad \partial^M \mathsf{W} \equiv \frac{\partial \mathsf{W}}{\partial \tilde{H}_M} = 2H^M \,,$$

Then, the FGK effective action can be written in the form

$$\begin{aligned} I_{\text{eff}}[H] \quad = \quad & \int d\tau \left\{ \tfrac{1}{2}\partial_M\partial_N \log \mathsf{W} \left(\dot{H}^M \dot{H}^N + \tfrac{1}{2}\mathcal{Q}^M \mathcal{Q}^N \right) \right. \\ & \left. + \left(\mathsf{W}^{-1}\dot{H}^M H_M \right)^2 + \left(\mathsf{W}^{-1}\mathcal{Q}^M H_M \right)^2 \right\} \,, \end{aligned}$$

All the information about the model is encoded in the Hessian potential $\mathsf{W}(H)$.

Having the $H^M(\tau)$ that solve this action, the black-hole solution is given by

$$e^{-2U(\tau)} = \mathsf{W}[H(\tau)] \,, \qquad Z^i(\tau) = \frac{\tilde{H}^i(H) + iH^i}{\tilde{H}^0(H) + iH^0} \,.$$

This shows that we can write all the static black-hole solutions of a given model $N = 2 \;\; d = 4$ supergravity exactly in the same way in terms of the functions $H^M(\tau)$.

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But these functions will be different for different solutions.

The equations of motion of the H^Ms are

$$
\begin{aligned}
-\tfrac{1}{2}\partial_M\partial_N \log \mathsf{W}\left(\dot{H}^M\dot{H}^N-\tfrac{1}{2}\mathcal{Q}^M\mathcal{Q}^N\right)+\left(\mathsf{W}^{-1}\dot{H}^M H_M\right)^2-(\mathsf{W}^{-1}\mathcal{Q}^M H_M)^2 &= {r_0}^2\,,\\
\tfrac{1}{2}\partial_M \log \mathsf{W}\left(\ddot{H}^M-{r_0}^2 H^M\right)+\left(\mathsf{W}^{-1}\dot{H}^M H_M\right)^2 &= 0\,,\\
\tfrac{1}{2}\partial_P\partial_M\partial_N \log \mathsf{W}\left[\dot{H}^M\dot{H}^N-\tfrac{1}{2}\mathcal{Q}^M\mathcal{Q}^N\right]+\partial_P\partial_M\log\mathsf{W}\,\ddot{H}^M & \\
-\frac{d}{d\tau}\left(\frac{\partial\Lambda}{\partial\dot{H}^P}\right)+\frac{\partial\Lambda}{\partial H^P} &= 0\,,
\end{aligned}
$$

with

$$\Lambda\equiv\left(\mathsf{W}^{-1}\dot{H}^M H_M\right)^2+(\mathsf{W}^{-1}\mathcal{Q}^M H_M)^2\,.$$

In the extremal case $r_0=0$ one sees immediately that $\dot{H}^P=\pm\frac{1}{\sqrt{2}}\mathcal{Q}^P$ satisfying the no-NUT condition $\dot{H}^P H_P=0$ solve all the equations.

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Actually, there are **two**: one for black holes and another for black strings.

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The theories

The scalar manifold of these theories is the hypersurface in "h-space"

$$\mathcal{V}(h) = C_{IJK} h^I h^J h^K = 1\,.$$

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The scalar manifold of these theories is the hypersurface in "h-space"

$$\mathcal{V}(h) = C_{IJK} h^I h^J h^K = 1\,.$$

If we then define the derived objects

$$h_I \equiv C_{IJK} h^J h^K\,, \qquad h^I_x \equiv -\sqrt{3}\,\frac{\partial h^I}{\partial \phi^x} \qquad \text{and} \qquad h_{Ix} \equiv \sqrt{3}\,\frac{\partial h_I}{\partial \phi^x}\,,$$

we can see that they satisfy the following relations

$$h^I h_I = 1 \qquad \text{and} \qquad h^I h_{Ix} = h_I h^I_x = 0\,.$$

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$$h^I h_I = 1 \qquad \text{and} \qquad h^I h_{Ix} = h_I h^I_x = 0\,.$$

The scalar metric g_{xy}, and the vector kinetic matrix, a_{IJ}, are given by

$$g_{xy} = h_{Ix} h^I_y \qquad \text{and} \qquad a_{IJ} = 3h_I h_J - 2C_{IJK} h^K = h_I h_J + h_{Ix} h^x_J\,.$$

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The bosonic action for $N=2$ $d=5$ supergravity with n vector supermultiplets is

$$\mathcal{I}_5 = \int_5 \left(R \star 1 + \tfrac{1}{2} g_{xy}\, d\phi^x \wedge \star d\phi^y - \tfrac{1}{2} a_{IJ} F^I \wedge \star F^J + \tfrac{1}{3\sqrt{3}} C_{IJK} F^I \wedge F^J \wedge A^K\right)\,.$$

The FGK formalisms for black holes and black strings

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This theory admits black-hole ($p = 0, \tilde{p} = 1$) and black strings ($p = 1, \tilde{p} = 0$) solutions. The corresponding metric ansätze are particular cases of the general one.

The FGK formalisms for black holes and black strings

This theory admits black-hole ($p=0, \tilde{p}=1$) and black strings ($p=1, \tilde{p}=0$) solutions. The corresponding metric ansätze are particular cases of the general one.

The effective action is

$$I_{\rm eff}[\tilde{U}, \phi^i] = \int d\tau \left\{ (\dot{\tilde{U}})^2 + \tfrac{(p+1)(\tilde{p}+2)}{3} g_{xy}\dot{\phi}^x \dot{\phi}^y - e^{2\tilde{U}} V_{\rm BB} + {r_0}^2 \right\} ,$$

where, in each case, we have to replace the black-brane potential $V_{\rm BB}$ by the the black-hole $V_{\rm bh}(\phi, q)$ and black-string potentials

$$\begin{cases} -V_{\rm bh}(\phi, q) & \equiv \quad a^{IJ} q_I q_J = {\mathcal{Z}_{\rm e}}^2 + 3\,\partial_x \mathcal{Z}_{\rm e}\,\partial^x \mathcal{Z}_{\rm e}\,, \\ -V_{\rm bs}(\phi, p) & \equiv \quad a_{IJ} p^I p^J = {\mathcal{Z}_{\rm m}}^2 + 3\,\partial_x \mathcal{Z}_{\rm m}\,\partial^x \mathcal{Z}_{\rm m}\,, \end{cases}$$

where we have defined the *electric and magnetic central charges* by

$$\mathcal{Z}_{\rm e}(\phi, q) \equiv h^I q_I\,, \qquad \qquad \mathcal{Z}_{\rm m}(\phi, p) \equiv h_I p^I\,.$$

8 – H-variables for black holes

We replace the original variables $\tilde{U}, \phi^x$ by new ones $\tilde{H}^I$ and H_I defined by

$$
\begin{aligned}
e^{-\tilde{U}/2} h^I(\phi) &\equiv \tilde{H}^I \,, \\
e^{-\tilde{U}} h_I(\phi) &\equiv H_I \,,
\end{aligned}
$$

and the new (unconstrained) function W

$$
\mathsf{W}(\tilde{H}) \equiv 2 C_{IJK} \tilde{H}^I \tilde{H}^J \tilde{H}^K \,.
$$

The homogeneity properties imply that

$$
\begin{aligned}
e^{-\frac{3}{2}\tilde{U}} &= \tfrac{1}{2}\mathsf{W}(H) \,, \\
h_I &= (\mathsf{W}/2)^{-2/3} H_I \,, \\
h^I &= (\mathsf{W}/2)^{-1/3} \tilde{H}^I \,.
\end{aligned}
$$

Changing the action to the H_I variables, it becomes

$$
-\tfrac{3}{2}\mathcal{I}[H] = \int d\rho \left[\partial^I \partial^J \log \mathsf{W} \left(\dot{H}_I \dot{H}_J + q_I q_J \right) - \tfrac{3}{2} {r_0}^2 \right] .
$$

9 – K-variables for black strings

We introduce two new sets of variables, K^I and $\tilde{K}_I$, related to the original ones $(\tilde{U}, \phi^x)$ by

$$
\begin{aligned}
e^{-\tilde{U}} h^I(\phi) &\equiv K^I \,, \\
e^{-2\tilde{U}} h_I(\phi) &\equiv \tilde{K}_I \,,
\end{aligned}
$$

and the new (unconstrained) function V

$$
\mathsf{V}(K) \equiv C_{IJK} K^I K^J K^K \,.
$$

The homogeneity properties imply that

$$
\begin{aligned}
e^{-3\tilde{U}} &= \mathsf{V}(K) \,, \\
h_I &= \mathsf{V}^{-2/3} \tilde{K}_I \,, \\
h^I &= \mathsf{V}^{-1/3} K^I \,.
\end{aligned}
$$

Changing the action to the K^I variables, it becomes

$$
-3\mathcal{I}[K] = \int d\rho \left[\partial_I \partial_J \log \mathsf{V} \left(\dot{K}^I \dot{K}^J + p^I p^J \right) - 3{r_0}^2 \right].
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Multiplying these equations by $\dot{H}_K$ we get $\dot{\mathcal{H}} = 0$, the Hamiltonian constraint

$$\mathcal{H} \equiv \partial^I \partial^J \log \mathsf{W} \left(\dot{H}_I \dot{H}_J - q_I q_J \right) + \tfrac{3}{2} {r_0}^2 = 0 \,,$$

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which is the equation of $\tilde{U}$ expressed in the new variables.

How useful are these new variables?

Non-extremal black holes and black branes

☞ In H-variables one immediately sees that, in the extremal case $r_0 = 0$

$$H_I = A_I \pm \rho q_I \, , \;\; \forall \, I \, ,$$

always solves the equations of motion in all theories.

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☞ A bit more difficult to see: in the extremal case $r_0 = 0$

$$H_I = A_I + \rho B_I \, ,$$

always solves all the equations of motion if

$$\partial^K V_{\rm bh}(B, q) = 0 \, .$$

(The scalars are always $\varphi_I = H_I/H_0$ and on the horizon $\varphi_I = B_I/B_0$).

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☞ The B_Is are called *fake charges*. Defining the *fake electric central charges*

$$\mathcal{Z}_{\rm e}(\phi,B) \equiv h^I B_I\,,$$

it is immediate to see that the following *first-order flow equations* are satisfied

$$\frac{de^{-\tilde{U}}}{d\rho} = \mathcal{Z}_{\rm e}(\phi,B)\,, \qquad\qquad \frac{d\phi^x}{d\rho} = -3e^{\tilde{U}}\partial^x \mathcal{Z}_{\rm e}(\phi,B)\,.$$

These first-order equations are extremely easy to obtain:

$$
\begin{aligned}
de^{-\tilde{U}} &= d(h^I h_I e^{-\tilde{U}}) \\
&= dh^I h_I e^{-\tilde{U}} + h^I d(h_I e^{-\tilde{U}}) \\
&= h^I d(h_I e^{-\tilde{U}}) \\
&= h^I dH_I \\
&= h^I B_I d\rho \\
&= \mathcal{Z}_{\mathrm{e}}(\phi, B) d\rho \,.
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Observe that the interest of these first-order equations is mainly formal since they are very difficult to integrate to obtain complete solutions.

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$$H_I = A_I \cosh r_0\rho + B_I \frac{\sinh r_0\rho}{r_0} \, .$$

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$$\begin{aligned} \partial^K\partial^I\partial^J \log \mathsf{W}(H)\left(B_I B_J - {r_0}^2 A_I A_J - q_I q_J\right) &= 0\,, \\ \partial^I\partial^J \log \mathsf{W}(H)\left(B_I B_J - {r_0}^2 A_I A_J - q_I q_J\right) &= 0\,. \end{aligned}$$

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☞ It is also possible to find all the non-extremal black holes with constant scalars of <u>all</u> the theories.

☞ Defining the new coordinate

$$\hat{\rho} \equiv \frac{\sinh(r_0 \rho)}{r_0 \cosh(r_0 \rho)}$$

we find the *first-order flow equations*

$$\frac{de^{-\tilde{U}}}{d\hat{\rho}} = \mathcal{Z}_{\mathrm{e}}(\phi, B)\,, \qquad\qquad \frac{d\phi^x}{d\hat{\rho}} = -3e^{\tilde{U}}\partial^x \mathcal{Z}_{\mathrm{e}}(\phi, B)\,.$$

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  We have proven that part of our ansatz is completely general, constructing a formalism ("H-FGK") that simplifies the construction of extremal and non-extremal (black-hole and also black-string solutions in $d = 5$.
- ★ We have shown the power of this approach finding very general solutions and results such as the *first-order flow equations* for extremal and non-extremal objects.

We are closer to determining the general form of all single, static, black-hole and black-string solutions of $N = 2$,$d = 4, 5$ theories.