The tensor hierarchy and supersymmetric domain walls of N=1,d=4 supergravity

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Work done in collaboration with *E. Bergshoeff, O. Hohm* (U. Groningen) *J. Hartong* (U. Bern) *M. Hübscher* and *P. Meessen* (IFT UAM/CSIC, Madrid)

Plan of the Talk:

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⇒ By using the embedding tensor method to gauge arbitrary 4-dimensional FTs, we may be able to find all their $(p+1)$-form potentials, their democratic formulations and the extended objects (branes) that can couple to them.

What we are going to do in this seminar:

[a] So far, only maximal and half-maximal supergravities have been studied from this point of view de Wit, Samtleben & Trigiante, arXiv:hep-th/0412173, Samtleben & Weidner arXiv:hep-th/0506237, Schon & Weidner, arXiv:hep-th/0602024, de Wit, Samtleben & Trigiante, arXiv:0705.2101, Bergshoeff, Gomis, Nutma & Roest, arXiv:0711.2035, de Wit, Nicolai & Samtleben, arXiv:0801.1294. The only exception is de Vroome & de Wit arXiv:0707.2717, but the $U(2)$ R-symmetry group has not been properly taken into account.

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2. We will find all the fields of the tensor hierarchy for arbitrary 4-dimensional FTs and we are going to construct a gauge -invariant action for all of them (democratic formulation).

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3. We will apply these results to $N = 1$ supergravity taking special care of the existence of $U(1)_R$ symmetry and a superpotential [a]. We will find all the $(p + 1)$-form potentials of $N = 1$ supergravity .
4. Only the 2- and 3-forms can be coupled to dynamic branes (strings and domain walls). We will construct a supersymmetric domain-wall effective action to be coupled to bulk $N = 1$ supergravity as sources and we will find the corresponding supersymmetric domain-wall solutions.

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2 – The embedding tensor method I: electric gaugings

Consider a general ($N = 1$ supergravity -inspired) 4-dimensional ungauged FT with bosonic fields $\{Z^i, A^\Lambda\}$ (gravity plays no relevant role here)

$$S_{\rm u}[Z^i, A^\Lambda] = \int \{-2\mathcal{G}_{ij^*} dZ^i \wedge \star dZ^{*\,j^*} - 2\Im{\rm m} f_{\Lambda\Sigma} F^\Lambda \wedge \star F^\Sigma + 2\Re{\rm e} f_{\Lambda\Sigma} F^\Lambda \wedge F^\Sigma - \star V_{\rm u}(Z, Z^*)\}\,.$$

with $F^\Lambda \equiv dA^\Lambda$, the fundamental (electric) field strengths and $f_{\Lambda\Sigma} = f_{\Lambda\Sigma}(Z)$.

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The action is invariant under the local Abelian transformations

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Let us assume this action is also invariant under the global transformations

$$\begin{aligned}
\delta_\alpha Z^i &= \alpha^A k_A{}^i(Z)\,,\\
\delta_\alpha f_{\Lambda\Sigma} &\equiv -\alpha^A \pounds_A f_{\Lambda\Sigma} = \alpha^A [T_{A\,\Lambda\Sigma} - 2T_{A\,(\Lambda}{}^\Omega f_{\Sigma)\Omega}]\,,\\
\delta_\alpha A^\Lambda &= \alpha^A T_{A\,\Sigma}{}^\Lambda A^\Sigma\,.
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Each embedding tensor $\vartheta_\Lambda{}^A$ defines a possible set of identifications:

$$\alpha^A(x) \equiv \Lambda^\Sigma \vartheta_\Sigma{}^A\,, \qquad\qquad A^A \equiv A^\Sigma \vartheta_\Sigma{}^A\,.$$

Leaving $\vartheta_\Lambda{}^A$ undetermined we can study all possibilities simultaneously.

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Now we construct derivatives $\mathfrak{D}$

$$\mathfrak{D}Z^i \equiv dZ^i + A^\Lambda \vartheta_\Lambda{}^A k_A{}^i\,,$$

covariant under

$$\begin{aligned}
\delta_\Lambda Z^i &= \Lambda^\Sigma \vartheta_\Sigma{}^A k_A{}^i(Z)\,,\\
\delta_\Lambda A^\Sigma &= -\mathfrak{D}\Lambda^\Sigma \equiv -(d\Lambda^\Sigma + \vartheta_\Lambda{}^A T_{A\,\Omega}{}^\Sigma A^\Lambda \Lambda^\Omega)\,.
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$\mathfrak{D}$ is covariant iff $\vartheta_\Lambda{}^A$ is an invariant tensor

$$\delta_\Lambda \vartheta_\Sigma{}^A = -\Lambda^\Omega Q_{\Omega\Sigma}{}^A = 0\,, \qquad\qquad Q_{\Sigma\Lambda}{}^A \equiv \vartheta_\Sigma{}^B T_{B\,\Lambda}{}^\Omega \vartheta_\Omega{}^A - \vartheta_\Sigma{}^B \vartheta_\Lambda{}^C f_{BC}{}^A\,.$$

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which satisfy the algebra

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Then we construct the covariant 2-form field strengths

$$F^\Lambda = dA^\Lambda + \tfrac{1}{2} X_{\Sigma\Omega}{}^\Lambda A^\Sigma \wedge A^\Omega\,,$$

and the gauge -invariant action of the electrically gauged FT takes the form

$$S_{\rm eg}[Z^i, A^\Lambda] = \int \{-2\mathcal{G}_{ij^*}\mathfrak{D}Z^i \wedge \star \mathfrak{D}Z^{*\,j^*} - 2\Im{\rm m} f_{\Lambda\Sigma} F^\Lambda \wedge \star F^\Sigma + 2\Re{\rm e} f_{\Lambda\Sigma} F^\Lambda \wedge F^\Sigma - \star V_{\rm eg}(Z, Z^*)\}$$

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- The theory (equations of motion) has more non-perturbative global symmetries that can be gauged . They include electric -magnetic duality rotations:

$$\begin{aligned}
\delta_\alpha Z^i &= \alpha^A k_A{}^i(Z)\,,\\
\delta_\alpha f_{\Lambda\Sigma} &= \alpha^A\{-T_{A\,\Lambda\Sigma} + 2T_{A\,(\Lambda}{}^{\Omega} f_{\Sigma)\Omega} - T_A{}^{\Omega\Gamma} f_{\Omega\Lambda} f_{\Gamma\Sigma}\}\,,\\
\delta_\alpha \begin{pmatrix} A^\Lambda \\ A_\Lambda \end{pmatrix} &= \alpha^A \begin{pmatrix} T_{A\,\Sigma}{}^{\Lambda} & T_A{}^{\Sigma\Lambda} \\ T_{A\,\Sigma\Lambda} & T_A{}^{\Sigma}{}_{\Lambda} \end{pmatrix} \begin{pmatrix} A^\Sigma \\ A_\Sigma \end{pmatrix}.
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Now we need to relate the α^{A} to the gauge parameters of the 1-forms Λ^{Λ} or Λ_{Λ} We need new (magnetic) components for the embedding tensor : $\vartheta^{\Lambda\, A}$. Then

$$\alpha^{A}(x) \equiv \Lambda^{\Sigma}\vartheta_{\Sigma}{}^{A} + \Lambda_{\Sigma}\vartheta^{\Sigma\, A}\,, \qquad A^{A} \equiv A^{\Sigma}\vartheta_{\Sigma}{}^{A} + A_{\Sigma}\vartheta^{\Sigma\, A}\,.$$

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Knowing (Gaillard & Zumino) that the T_A matrices either belong to $\mathfrak{sp}(2n_V,\mathbb{R})$ or vanish, we introduce the symplectic notation

$$A^M \equiv \begin{pmatrix} A^\Sigma \\ A_\Sigma \end{pmatrix} \qquad \vartheta_M{}^A \equiv (\vartheta_\Sigma{}^A, \vartheta^{\Sigma\, A}) \qquad \alpha^A(x) \equiv \Lambda^M \vartheta_M{}^A\,,$$

$$(T_{A\, M}{}^N) \equiv \begin{pmatrix} T_{A\,\Sigma}{}^\Lambda & T_A{}^{\Sigma\Lambda} \\ & \\ T_{A\,\Sigma\Lambda} & T_A{}^\Sigma{}_\Lambda \end{pmatrix}.$$

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The electric and magnetic charges must be *mutually local* (de Wit, Samtleben & Trigiante, arXiv:`hep-th/0507289`) satisfying the second quadratic constraint:

$$Q^{AB} \equiv \tfrac{1}{4}\vartheta^{M\,A}\vartheta_M{}^B = 0\,.$$

Now we can repeat the procedure of the electric case:

First we construct derivatives $\mathfrak{D}$

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which only works if $\vartheta_M{}^A$ is an invariant tensor

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Before moving forward, we must impose another constraint on the embedding tensor on top of the two quadratic ones $Q_{MN}{}^A = Q^{AB} = 0$:

$$L_{MNP} \equiv X_{(MNP)} = \vartheta_{(M}{}^A T_{A\,NP)} = 0 \,.$$

This *linear* or *representation constraint* is based on supergravity and eliminates certain possible representations of the embedding tensor . On the other hand, we cannot construct gauge -covariant 2-form field strengths F^M without it!

4 – The 4-d tensor hierarchy

To construct the gauge -covariant 2-form field strengths F^M we take the covariant derivative of the gauge -covariant "field strength" $\mathcal{D}Z^i$:

$$\mathfrak{D}\mathfrak{D}Z^i = [dA^M + \tfrac{1}{2}X_{NP}{}^M A^N \wedge A^P]\vartheta_M{}^A k_A{}^i \,,$$

which suggests the definition

$$F^M \equiv dA^M + \tfrac{1}{2}X_{NP}{}^M A^N \wedge A^P + \Delta F^M \,, \qquad \vartheta_M{}^A \Delta F^M = 0 \,,$$

so we have the Bianchi identity

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But we do not need it to move forward.

If we take the covariant derivative of the gauge -covariant 2-form field strength F^{M} we find

$$\mathfrak{D}F^{M} = Z^{MA}\{\mathfrak{D}B_{A} + T_{A\,RS}A^{R} \wedge [dA^{S} + \tfrac{1}{3}X_{NP}{}^{S}A^{N} \wedge A^{P}]\}\,.$$

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To determine $\Delta G_C{}^M$ we need to find invariant tensors that vanish upon contraction with $Y_{AM}{}^C$. They appear automatically when we take the gauge -covariant derivative of the Bianchi identity and $G_C{}^M$ (if we “forget” we are in 4 dimensions!).

Acting with $\mathfrak{D}$ on the Bianchi identity of H_A we find

$$Y_{AM}{}^{C}\{\mathfrak{D}G_{C}{}^{M} - F^{M} \wedge H_{A}\} = 0\,, \;\Rightarrow\; \mathfrak{D}G_{C}{}^{M} = F^{M} \wedge H_{A} + \Delta\mathfrak{D}G_{C}{}^{M}\,,$$

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This implies that there are 3 such tensors $W_C{}^{MAB}, W_{CNPQ}{}^{M}, W_{CNP}{}^{EM}$ that vanish contracted with $Y_{AM}{}^{C}$ and which we can use to build $\Delta G_C{}^{M}$.

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The natural solution is

$$\Delta G_{C}{}^{M} = W_{C}{}^{MAB}D_{AB} + W_{CNPQ}{}^{M}D^{NPQ} + W_{CNP}{}^{EM}D_{E}{}^{NP}\,,$$

and $\delta_{\Lambda}D_{AB}, \delta_{\Lambda}D^{NPQ}, \delta_{\Lambda}D_{E}{}^{NP}$ will follow from the gauge -covariance of $G_{C}{}^{M}$.

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But, what does it mean?
What is the meaning of the additional fields?

5 – The meaning of the $d = 4$ tensor hierarchy

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- These two duality relations together with the Bianchi identity $\mathfrak{D}F^M = Z^{MA}H_A$ give a set of electric -magnetic duality -covariant Maxwell equations:

$$\mathfrak{D}F^\Lambda = -\tfrac{1}{4}\vartheta_\Lambda{}^A \star j_A \quad , \qquad \mathfrak{D}G_\Lambda = \tfrac{1}{4}\vartheta^{\Lambda\, A} \star j_A \quad .$$

☞ The 3-forms $C_C{}^M$ must be "*dual to constants*", i.e. to the deformation parameters. Their indices are indeed conjugate to those of the embedding tensor $\vartheta_M{}^C$. This duality is expressed through the formula

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This equation is similar to the consistency condition (gauge or Noether identity) that Noether currents must satisfy off-shell in FTs with gauge invariance:

$$\mathfrak{D} \star j_A = -2(k_A{}^i \mathcal{E}_i + \text{c.c.}) + 4T_{A\,MN} G^M \wedge G^N + \star Y_A{}^{MC} \frac{\partial V}{\partial \vartheta_M{}^C} \ ,$$

where $\mathcal{E}_i$ is the e.o.m. of Z^i. Both equations, together, imply

$$k_A{}^i \mathcal{E}_i + \text{c.c.} = 0 \ ,$$

which is equivalent to the scalar e.o.m. for symmetric σ-models.

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To confirm this interpretation we must construct a gauge -invariant (*democratic*) action for **all** these fields, (including the embedding tensor $\vartheta_M{}^A(x)$!).

This gauge -invariant action is given by

$$S[g_{\mu\nu}, Z^i, A^M, B_A, C_A{}^M, D_E{}^{NP}, D_{AB}, D^{MNP}, \vartheta_M{}^A] =$$

$$\int \{-2\mathcal{G}_{ij^*}\mathfrak{D}Z^i \wedge \star\mathfrak{D}Z^{*\,j^*} + 2F^\Sigma \wedge G_\Sigma - \star V$$

$$-4Z^{\Sigma A}B_A \wedge \left(F_\Sigma - \tfrac{1}{2}Z_\Sigma{}^B B_B\right) - \tfrac{4}{3}X_{[MN]\Sigma}A^M \wedge A^N \wedge \left(F^\Sigma - Z^{\Sigma B}B_B\right)$$

$$-\tfrac{2}{3}X_{[MN]}{}^\Sigma A^M \wedge A^N \wedge \left(dA_\Sigma - \tfrac{1}{4}X_{[PQ]\Sigma}A^P \wedge A^Q\right)$$

$$-2\mathfrak{D}\vartheta_M{}^A \wedge (C_A{}^M + A^M \wedge B_A)$$

$$+2Q_{NP}{}^E(D_E{}^{NP} - \tfrac{1}{2}A^N \wedge A^P \wedge B_E) + 2Q^{AB}D_{AB} + 2L_{MNP}D^{MNP}\} \,.$$

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We are going to review ungauged $N = 1$ supergravity and its global symmetries and then we are going to gauge them using the embedding tensor formalism.

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All fermions are represented by chiral 4-component spinors:

$$\gamma_5 \psi_{\mu} = -\psi_{\mu}\,, \text{etc.}$$

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The spinors transform as *sections* of the bundle: under Kähler transformations

$$\delta_\lambda \mathcal{K} = \lambda(Z) + \lambda^*(Z^*)\,, \qquad \delta_\lambda \psi_\mu = -\tfrac{1}{4}[\lambda(Z) - \lambda^*(Z^*)]\psi_\mu\,,$$

and their covariant derivatives contain the pullback of the Kähler connection 1-form $\mathcal{Q} \equiv \mathcal{Q}_i dZ^i + \mathcal{Q}_{i^*} dZ^{*i^*}$ e.g.

$$\mathcal{D}_\mu \psi_\nu = \{\nabla_\mu + \tfrac{i}{2}\mathcal{Q}_\mu\}\psi_\nu\,.$$

$N=1$ supergravity allows for an arbitrary holomorphic kinetic matrix $f_{\Lambda\Sigma}(Z)$ for the vector fields which occurs in the action in the terms

$$-2\Im\mathrm{m} f_{\Lambda\Sigma} F^{\Lambda} \wedge \star F^{\Sigma} + 2\Re\mathrm{e} f_{\Lambda\Sigma} F^{\Lambda} \wedge F^{\Sigma}\,, \qquad\qquad F^{\Lambda} \equiv dA^{\Lambda}\,.$$

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Finally, ungauged $N = 1$ supergravity allows for a holomorphic superpotential $\mathcal{W}(Z)$ which appears through the covariantly holomorphic section of Kähler weight $(1, -1)$ $\mathcal{L}(Z, Z^*)$:

$$\mathcal{L}(Z, Z^*) = \mathcal{W}(Z) e^{\mathcal{K}/2}\,, \qquad \mathcal{D}_{i^*} \mathcal{L} = 0\,,$$

which couples to the fermions in various ways and gives rise to the scalar potential

$$V_{\mathrm{u}}(Z, Z^*) = -24|\mathcal{L}|^2 + 8\mathcal{G}^{ij^*} \mathcal{D}_i \mathcal{L} \mathcal{D}_{j^*} \mathcal{L}^*\,.$$

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$$\begin{aligned} S_{\mathrm{u}}[g_{\mu\nu}, Z^i, A^{\Lambda}] \;=\; & \int \{\star R - 2\mathcal{G}_{ij^*} dZ^i \wedge \star dZ^{*\,j^*} - 2\Im\mathrm{m} f_{\Lambda\Sigma} F^{\Lambda} \wedge \star F^{\Sigma} \\ & + 2\Re\mathrm{e} f_{\Lambda\Sigma} F^{\Lambda} \wedge F^{\Sigma} - \star V_{\mathrm{u}}(Z,Z^*)\}\,. \end{aligned}$$

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Main difference with the general case: the existence of $H_{\rm aut} = U(1)_R$.

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- ☞ The superpotential $\mathcal{L}(Z, Z^*)$ is not a fundamental field and this phase change is not a symmetry unless it can be reabsorbed into a transformation of the scalars.
- ☞ But this would mean that we are dealing with a $A = \mathrm{a}$ symmetry and we can say that a non-vanishing superpotential breaks $U(1)_R$ and we cannot gauge it.

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$$\mathcal{L} \neq 0\,,\quad \Rightarrow \vartheta_M{}^A(\delta_A{}^{\underline{a}}\mathcal{P}_{\underline{a}} + \delta_A{}^{\#}\mathcal{P}_\#) = 0\ .$$

9 – The $N = 1, d = 4$ bosonic tensor hierarchy

We have found that, for non-vanishing superpotential , the embedding tensor must satisfy another constraint of purely fermioninc origin

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This will happen in $N = 1$ supergravity if we find new Stückelberg shifts

$$\delta' B_A \sim \delta_h B_A + Y_A \Lambda \quad \text{and} \quad \delta' C_C{}^M = \delta_h C_C{}^M + Y_C \Lambda^M\,.$$

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Observe that we are going to obtain, independently, the gauge transformations of the fields, confirming or refuting the hierarchy's results.

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$$\delta_\eta \chi^i = i\,\not{\mathfrak{D}} Z^i \eta^* + 2\mathcal{G}^{ij^*}\mathcal{D}_{j^*}\mathcal{L}^*\eta\,, \qquad \mathfrak{D} Z^i = dZ^i + A^M \vartheta_M{}^A k_A{}^i\,.$$

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We find the expected result

$$\begin{aligned}
[\delta_\eta\,,\,\delta_\epsilon]Z^i &= \delta_{\text{g.c.t.}}Z^i + \delta_h Z^i\,,\\
\delta_{\text{g.c.t.}}Z^i &= \pounds_\xi Z^i = +\xi^\mu\partial_\mu Z^i\,,\\
\delta_h Z^i &= \Lambda^M\vartheta_M{}^A k_A{}^i\,,\\
\xi^\mu &\equiv \tfrac{i}{4}(\bar{\epsilon}\gamma^\mu\eta^* - \bar{\eta}\gamma^\mu\epsilon^*)\,,\\
\Lambda^M &\equiv \xi^\mu A^M{}_\mu\,.
\end{aligned}$$

The 1-forms A^M

We introduce supersymmetric partners λ_Σ for the magnetic 1-forms A_Σ and make the symplectic -covariant *Ansatz*

$$\begin{aligned} \delta_\epsilon A^M{}_\mu &= -\tfrac{i}{8}\bar{\epsilon}^*\gamma_\mu\lambda^M + \text{c.c.}\,, \\ \delta_\epsilon \lambda^M &= \tfrac{1}{2}\left[\not{F}^{M+} + i\mathcal{D}^M\right]\epsilon\,, \end{aligned}$$

where we have defined the symplectic vector

$$\mathcal{D}^M \equiv \begin{pmatrix} \mathcal{D}^\Lambda \\ \mathcal{D}_\Lambda \end{pmatrix} \equiv \begin{pmatrix} \mathcal{D}_\Lambda \\ f_{\Lambda\Sigma}\mathcal{D}^\Sigma \end{pmatrix}\,, \qquad \mathcal{D}^\Lambda = -\Im\text{m} f^{\Lambda\Sigma}\,(\vartheta_\Sigma{}^A + f^*_{\Sigma\Omega}\vartheta^{\Omega\,A})\mathcal{P}_A\,.$$

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We introduce the supersymmetric partners of the 2-forms $B_{A\,\mu\nu}$ ζ_A, φ_A (linear supermultiplets)

$$\delta_\epsilon \zeta_A = -i\left[\tfrac{1}{12}\not{H}'_A + \not{\mathfrak{D}}\varphi_A\right]\epsilon^* - 4\delta_A{}^{\mathbf{a}}\varphi_{\mathbf{a}}\mathcal{L}^*\epsilon\,,$$

$$\delta_\epsilon B_{A\,\mu\nu} = \tfrac{1}{4}\left[\bar{\epsilon}\gamma_{\mu\nu}\zeta_A + \text{c.c.}\right] - i\left[\varphi_A\bar{\epsilon}^*\gamma_{[\mu}\psi_{\nu]} - \text{c.c.}\right] + 2T_{A\,MN}A^M{}_{[\mu}\delta_\epsilon A^N{}_{\nu]}\,,$$

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This corresponds to th manifestly symplectic -invariant scalar potential

$$V_{\text{e-mg}} = V_{\text{u}} - \tfrac{1}{2}\Re\text{e}\,\mathcal{D}^M\vartheta_M{}^A\mathcal{P}_A = V_{\text{u}} + \tfrac{1}{2}\mathcal{M}^{MN}\vartheta_M{}^A\vartheta_N{}^B\mathcal{P}_A\mathcal{P}_B\,,$$

where

$$(\mathcal{M}^{MN}) \equiv \begin{pmatrix} I^{\Lambda\Sigma} & I^{\Lambda\Omega}R_{\Omega\Sigma} \\ R_{\Lambda\Omega}I^{\Omega\Sigma} & I_{\Lambda\Sigma} + R_{\Lambda\Omega}I^{\Omega\Gamma}R_{\Gamma\Sigma} \end{pmatrix}, \qquad \begin{aligned} f_{\Lambda\Sigma} &\equiv R_{\Lambda\Sigma} + iI_{\Lambda\Sigma}\,, \\ I^{\Lambda\Omega}I_{\Omega\Sigma} &\equiv \delta^\Lambda{}_\Sigma\,. \end{aligned}$$

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We actually find a complex 3-form $\mathcal{C}_{\mu\nu\rho} = C^1{}_{\mu\nu\rho} + iC^2{}_{\mu\nu\rho}$ with supersymmetry transformations

$$\delta_\epsilon \mathcal{C}_{\mu\nu\rho} = 12i\mathcal{L}\,\bar{\epsilon}^* \gamma_{[\mu\nu}\psi^*{}_{\rho]} + 2\mathcal{D}_i\mathcal{L}\bar{\epsilon}^*\gamma_{\mu\nu\rho}\chi^i + \text{c.c.}$$

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$$d\mathcal{C} = (\mathbf{g^1} + i\mathbf{g^2}) \star (-24|\mathcal{L}|^2 + 8\mathcal{G}^{ij^*}\mathcal{D}_i\mathcal{L}\mathcal{D}_{j^*}\mathcal{L}^*)\,, \quad \text{or} \quad dC^i = \tfrac{1}{2}\star\frac{\partial V}{\partial \mathbf{g^i}}\,,\ \ i = 1, 2\,.$$

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There is always a 3-form for each deformation parameter.

The 3-form that appears in the 2-form field strengths happens to be

$$C = \tfrac{1}{2}(\mathbf{g^1}C^2 - \mathbf{g^2}C^1)\,.$$

The 4-forms $D_{AB}, D^{NPQ}, D_E{}^{NP}, D^M$

We only check the closure of the local supersymmetry algebra in the ungauged $\vartheta_M{}^A = 0$ case when there are no symmetries acting on the 1-forms i.e. $T_{A\,M}{}^N = 0$ for simplicity.

The supersymmetry transformations are

$$
\begin{aligned}
\delta_\epsilon D_{AB} &= -\tfrac{i}{2} \star \mathcal{P}_{[A}\partial_i \mathcal{P}_{B]}\bar{\epsilon}\chi^i + \text{c.c.} - B_{[A} \wedge \delta_\epsilon B_{B]}\,, \\
\delta_\epsilon D^{NPQ} &= 10 A^{(N} \wedge F^P \wedge \delta_\epsilon A^{Q)}\,, \\
\delta_\epsilon D_E{}^{NP} &= C_E{}^P \wedge \delta_\epsilon A^N\,. \\
\delta_\epsilon D^M &= -\tfrac{i}{2} \star \mathcal{L}^* \bar{\epsilon}\lambda^M + \text{c.c.} + C \wedge \delta_\epsilon A^M\,.
\end{aligned}
$$

This proves that D^M can be consistently added to the supersymmetric theory. Its role in the action will be that of Lagrange multiplier of the constraint Q_M.

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We are going to focus on the domain walls associated to the 3-form C^{1} $(\mathbf{g^2}=0)$. We consider the ungauged theory with only chiral supermultiplets and superpotential

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The metric of a 4-d domain-wall solution can always be written in the form

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If the $Z^i = Z^i(y)$ the gravitino Killing spinor equation $\delta_\epsilon\psi_\mu = 0$ is be solved by

$$(e^{-i\alpha/2}\epsilon) \pm i\gamma^{012}(e^{-i\alpha/2}\epsilon)^* = 0\,, \qquad e^{i\alpha} \equiv \mathcal{L}/|\mathcal{L}|\,.$$

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The first-order *flow equations* imply the second-order supergravity e.o.m..

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In the static gauge $\partial X^\mu/\partial\xi^m = \delta^\mu{}_m$ it can be seen that this action is invariant to lowest order in fermions under the supersymmetry transformations of $g_{\mu\nu}, Z^i, C'_{\mu\nu\rho}$ if the spinors satisfy the BPS domain-wall projection $(e^{-i\alpha/2}\epsilon) \pm i\gamma^{012}(e^{-i\alpha/2}\epsilon)^* = 0$.

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Thus, we consider the bulk supergravity action,

$$S_{\text{bulk}} = \frac{1}{\kappa^2}\int d^4x\sqrt{|g|}\left[R + 2\mathcal{G}_{ij^*}\partial_\mu Z^i \partial^\mu Z^{*\,j^*} - \mathbf{g}^2(x)V(Z,Z^*) - \frac{1}{3\sqrt{|g|}}\epsilon^{\mu\nu\rho\sigma}\partial_\mu \mathbf{g}(x) C_{\nu\rho\sigma}\right]$$

14 – Sourceful domain-wall solutions of $N = 1$ supergravity

To couple our candidate to domain-wall source to the bulk $N = 1$ supergravity action we need to introduce consistently C into it by

1. Rescaling $\mathcal{L} \longrightarrow \mathbf{g}\mathcal{L}$ ($\mathbf{g} \equiv \mathbf{g}^{\mathbf{1}}$) everywhere **except at the 3-form supersymmetry transformation rule**.

2. Promoting the constant $\mathbf{g}$ to a scalar field $\mathbf{g}(x)$ and adding to the bulk supergravity action a Lagrange -multiplier term containing the 3-form to enforce the constancy of $\mathbf{g}$.

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and the brane source action

$$S_{\text{brane}} = -\int d^4x\, \mathbf{f}(y)\left\{|\mathcal{L}|\sqrt{|g_{(3)}|} \pm \tfrac{1}{4!}\epsilon^{mnp} C_{\underline{mnp}}\right\},$$

where $\mathbf{f}(y)$ is a distribution function of the domain walls' common transverse direction $x^3 \equiv y$: $\mathbf{f}(y) = \delta^{(1)}(y - y_0)$ for a single domain wall placed at $y = y_0$ etc.

The equations of motion that follow from $S \equiv S_{\text{bulk}} + S_{\text{brane}}$ are

$$
\begin{aligned}
\mathcal{E}_{\mathbf{g}}^{\mu\nu} &= -\frac{\kappa^2}{2}\mathbf{f}(y)|\mathcal{L}|\frac{\sqrt{|g_{(3)}|}}{\sqrt{|g|}}g_{(3)}^{mn}\delta_m{}^{\mu}\delta_n{}^{\nu}\,,\\
\mathcal{G}^{ij^*}\mathcal{E}_{\mathbf{g}\,i^*} &= -\frac{\kappa^2}{8}\mathbf{f}(y)\frac{\sqrt{|g_{(3)}|}}{\sqrt{|g|}}e^{i\alpha}\mathcal{N}^i\,,\\
\epsilon^{\mu\nu\rho\sigma}\partial_\sigma\mathbf{g}(x) &= \pm\frac{\kappa^2}{8}\mathbf{f}(y)\epsilon^{mnp}\delta_m{}^{\mu}\delta_n{}^{\nu}\delta_p{}^{\rho}\,,\\
\epsilon^{\mu\nu\rho\sigma}\partial_\mu C_{\nu\rho\sigma} &= 6\mathbf{g}(x)V(Z,Z^*)\,,
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where $\mathcal{E}_{\mathbf{g}}^{\mu\nu}$ and $\mathcal{E}_{\mathbf{g}\,i^*}$ are the Einstein and scalar equations of motion with $\mathbf{g}(x) \neq 0$.

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The third equation is that of the 3-form and is solved if $\mathbf{g}$ is a function of y satisfying

$$\partial_{\underline{y}}\mathbf{g} = \pm\tfrac{1}{8}\kappa^2\mathbf{f}(y)\,.$$

$\mathbf{g}(y)$ will have step-like discontinuities at the locations of the domain walls.

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The fourth equation ($\mathbf{g}(x)$') states that C is the dual of the scalar potential.

It can now be checked that the Einstein and scalar equations of motion with sources are identically satisfied if $H(y)$ and the scalars $Z^i(y)$ satisfy the *sourceful flow equations*

$$\begin{aligned} \partial_{\underline{y}} Z^i &= \pm \mathbf{g}(y) e^{i\alpha} \mathcal{N}^i H^{1/2}\,, \\ \partial_{\underline{y}} H^{-1/2} &= \pm 2\mathbf{g}(y) |\mathcal{L}|\,, \end{aligned}$$

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which can be derived from the modified fermion supersymmetry transformations

$$\begin{aligned} \delta_\epsilon \psi_\mu &= \mathcal{D}_\mu \epsilon + i\mathbf{g}(x)\mathcal{L}\gamma_\mu \epsilon^* \,, \\ \delta_\epsilon \chi^i &= i \not{\partial} Z^i \epsilon^* + \mathbf{g}(x) \mathcal{N}^i \epsilon \,. \end{aligned}$$

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A fully supersymmetric "democratic" formulation of $N = 1 \quad d = 4$ supergravity including all higher-rank forms and local coupling constants $\vartheta_M{}^A(x), \mathbf{g}^1(x), \mathbf{g}^2(x)$ is necessary to accomodate these modifications.

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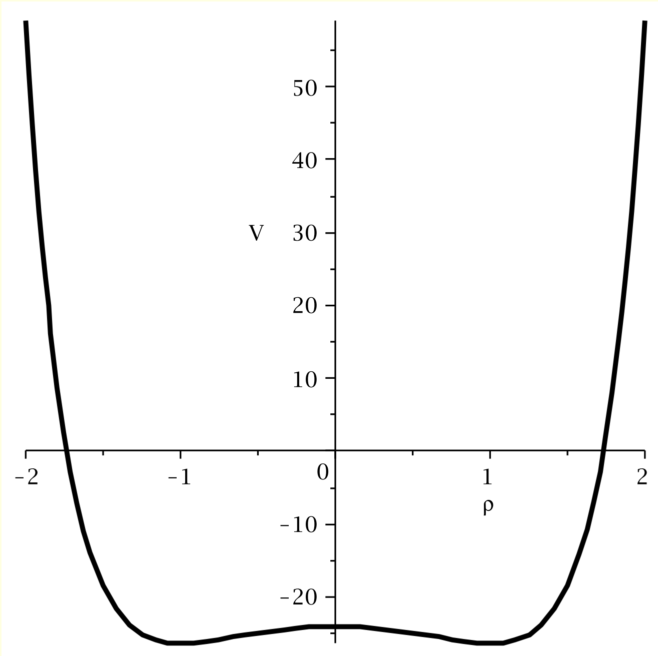
Observe that the space-dependent coupling constant $\mathbf{g}(x)$, sourced by domain walls , may modify the effective scalar potential dramatically.

15 – A simple example

Let us consider the model (1 chiral multiplet) defined by

$$\mathcal{K} = |Z|^2\,, \qquad \mathcal{W} = 1\,, \qquad (\mathcal{L} = e^{|Z|^2/2}\,,\ \mathcal{N}^Z = 2Z^* e^{|Z|^2/2})\,.$$

These choices lead to the Mexican-hat-type potential $V = -8(3-\rho^2)e^{\rho^2/2}$ $(\rho \equiv |Z|)$ with a maximum and degenerate minimum at $\rho = 0$ and $\rho = +1$ resp. with $V(0) = -24$ and $V(1) = -16\sqrt{e} \sim -26.4$.



The *sourceful flow equations* take the form ($\mathrm{Arg}\, Z = \mathrm{const}$)

$$\begin{aligned} \partial_{\underline{y}}\rho &= \pm 2\mathbf{g}(y)\rho e^{\rho^2/2} H^{1/2}\,, \\ \partial_{\underline{y}} H^{-1/2} &= \pm 2\mathbf{g}(y) e^{\rho^2/2}\,. \end{aligned}$$

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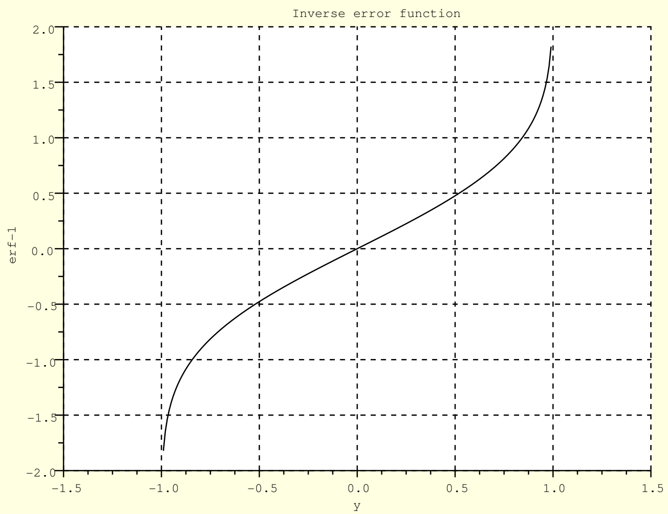
II-b Solutions with $\mathbf{g} \neq 0$ and $\partial_{\underline{y}} Z \neq 0$:

$$H = c/\rho^2\,,$$

$$\rho = \sqrt{2}\,\mathrm{erf}^{-1}\left[\mathbf{G}(y)\right]\,, \qquad \mathbf{G}(y) \equiv \pm\sqrt{\tfrac{8c}{\pi}}\int \mathbf{g}(y)dy + d\,.$$

erf^{-1} is the inverse of the *normalized error function*

$$\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du = -\mathrm{erf}(-x)\,.$$



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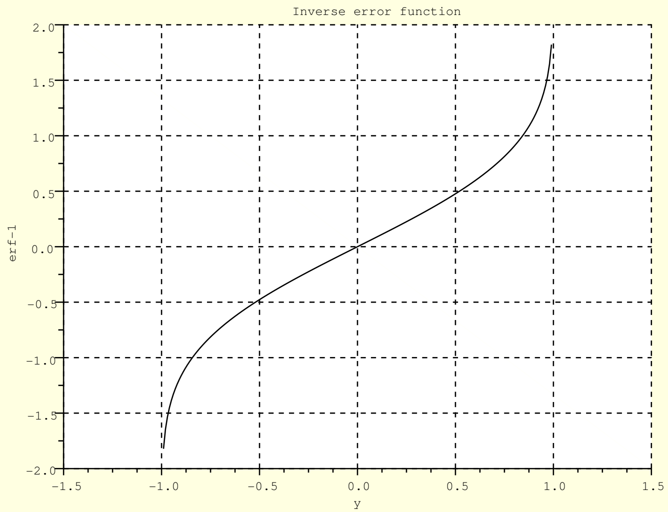


Then $\mathbf{G}(y) \in [0,1)$, which, for constant $\mathbf{g}$ requires that we cut the spacetime at finite values of y. To have more general $\mathbf{g}(y)$ or to do the custs consistently **we have to add sources**.

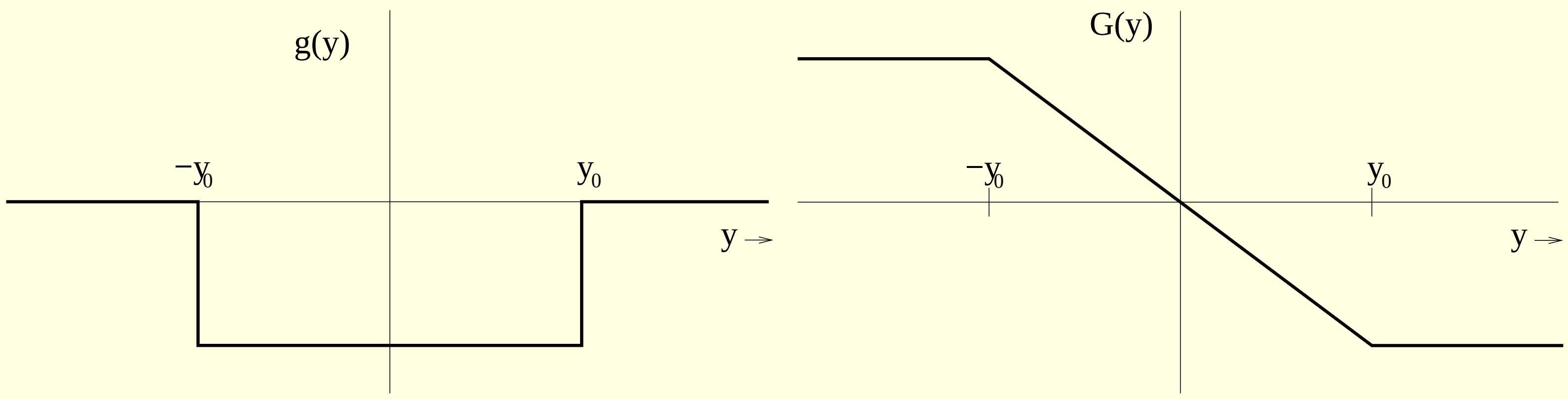
Let us consider, first, a single, infinitely thin domain-wall source of tension $q > 0$ placed at $y = y_0$:

$$\mathbf{f}(y) = q\delta(y-y_0)\,, \qquad \mathbf{g}(y) = \pm\frac{\kappa^2 q}{16}[\theta(y-y_0)-\theta(y_0-y)]\,, \qquad \mathbf{G}(y) = \frac{\sqrt{c}\kappa^2 q}{\sqrt{32\pi}}|y-y_0|+d\,.$$

$\mathbf{G}(y)$ is always unbounded and the solution is not well defined unless we cut the space by hand.

A possible solution: we introduce two parallel domain walls with opposite tension (a Randall-Sundrum-like construction) and charge at a different point ($y = -y_0$ with $y_0 > 0$ for simplicity) so

$$\begin{aligned} \mathbf{f}(y) &= q\delta(y - y_0) - q\delta(y + y_0)\,, \\ \mathbf{g}(y) &= \pm\frac{\kappa^2 q}{16}[\theta(y - y_0) - \theta(y_0 - y) - \theta(y + y_0) + \theta(-y_0 - y)]\,, \\ \mathbf{G}(y) &= \sqrt{\frac{c}{32\pi}}\kappa^2 q\,(|y - y_0| - |y + y_0|) + d\,. \end{aligned}$$



Choosing $d = \sqrt{\frac{c}{8\pi}}\kappa^2 q y_0$ we can set $\mathbf{G}(+\infty) = \mathbf{G}(+y_0) = 0$ and $\rho(y) = \rho(+y_0) = 0$ for $y > y_0$.

In the interior of the $\mathbf{g}(y) \neq 0$ region ρ approaches zero as $\rho \sim \frac{1}{4}\sqrt{c}\kappa^2 q(y_0 - y)$ so the metric approaches AdS_4

$$H \sim \frac{R^2}{(y_0 - y)^2}\,, \qquad R = \frac{4}{\kappa^2 q}\,.$$

The value $\mathbf{G}(-y_0) = \sqrt{\frac{c}{2\pi}}\kappa^2 q y_0 = \mathbf{G}(-\infty)$, can be tuned by varying distance between the domain-wall sources (y_0). It has to be smaller or equal than 1.

If $\mathbf{G}(-y_0) < 1$ then $\rho(-y_0)$ is finite and ρ approaches $y = -y_0$ from the interior of the $\mathbf{g}(y) \neq 0$ region as

$$\rho \sim -\sqrt{\frac{c}{2\pi}}\frac{\kappa^2 q}{\mathrm{erf}'[\rho(-\infty)/\sqrt{2}]}(y + y_0)\,,$$

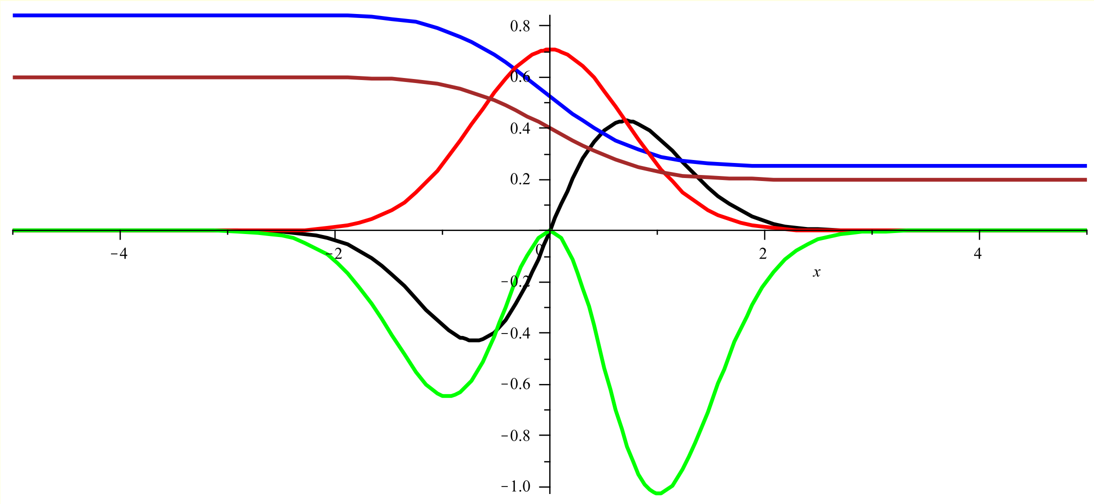
so the metric approaches another AdS_4 region.

This solution we have obtained smoothly interpolates between two AdS_4 regions one of which (the $\rho = 0$ one) corresponds to a supersymmetric vacuum of the theory.

The two infinitely-thin domain-wall sources setup can be understood as a crude approximation to the following configuration with domain-wall sources of finite thickness

$$\mathbf{f}(y) = qye^{-y^2}\,, \qquad \mathbf{g}(y) = \mp\frac{\kappa^2 q}{16}e^{-y^2}\,, \qquad \mathbf{G}(y) = -\frac{\kappa^2 q\sqrt{c}}{8}\mathrm{erf}(y) + d\,.$$

in which $\mathbf{g}(y)$ only vanishes asymptotically.



The profiles of some of the functions ocurring in this solution: the **black line**: the source, $\mathbf{f}(y)$, **red line**: the coupling constant $\mathbf{g}(y)$, **brown line** $\mathbf{G}(y)$, **blue line**: the scalar $\rho(y)$, **green line**: the effective potential as seen by the solution, *i.e.* $\mathbf{g^2(y)}V$. Observe that the degeneracy is removed by the sources.

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- We have seen that in some cases domain-wall sources have to be introduced to construct sensible domain-wall solutions. These sources introduce a spacetime-dependent coupling constant $\mathbf{g}(x)$ that can have dramatic effects on the form of the solutions.