IIB 9-BRANES

Tomás Ortín (I.F.T., Madrid)

Seminar given on February 9th 2006 at the Workshop on Gravitational Aspects of Strings and Branes

Based on hep-th/0601128 and on work in preparation. Work done in collaboration with

Eric Bergshoeff, Mees de Roo, Sven Kerstan (U. Groningen, The Netherlands)
and
Fabio Riccioni (U. Cambridge, U.K.)

Plan of the Talk:

1 − $N = 2B, d = 10$ SUEGRA Revisited

[a] E.A. Bergshoeff, M. de Roo, B. Janssen and T. Ortín, *Nucl. Phys.* **B550** (1999) 289. `hep-th/9901055`. E.A. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *Class. Quant. Grav.* **18** (2001) 3359. `hep-th/0103233`.

[b] E.A. Bergshoeff, M. de Roo, S.F. Kerstan and F. Riccioni, *JHEP* **0508** (2005) 098. `hep-th/0506013`.

1 − $N = 2B, d = 10$ SUEGRA Revisited

It has long been known that $N = 2B, d = 10$ SUEGRA must contain a RR 10-form potential $A_{(10)}$ associated to D9-branes (T-duality, κ-symmetry, susy algebra...).

[a] E.A. Bergshoeff, M. de Roo, B. Janssen and T. Ortín, *Nucl. Phys.* **B550** (1999) 289. `hep-th/9901055`. E.A. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *Class. Quant. Grav.* **18** (2001) 3359. `hep-th/0103233`.

[b] E.A. Bergshoeff, M. de Roo, S.F. Kerstan and F. Riccioni, *JHEP* **0508** (2005) 098. `hep-th/0506013`.

1 − $N = 2B, d = 10$ SUEGRA Revisited

It has long been known that $N = 2B, d = 10$ SUEGRA must contain a RR 10-form potential $A_{(10)}$ associated to D9-branes (T-duality, κ-symmetry, susy algebra...).

The $A_{(10)}$ potential gauge and supersymmetry transformations must be consistent with those of the other fields[a]

[a] E.A. Bergshoeff, M. de Roo, B. Janssen and T. Ortín, *Nucl. Phys.* **B550** (1999) 289. hep-th/9901055. E.A. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *Class. Quant. Grav.* **18** (2001) 3359. hep-th/0103233.

[b] E.A. Bergshoeff, M. de Roo, S.F. Kerstan and F. Riccioni, *JHEP* **0508** (2005) 098. hep-th/0506013.

1 − $N = 2B, d = 10$ SUEGRA Revisited

It has long been known that $N = 2B, d = 10$ SUEGRA must contain a RR 10-form potential $A_{(10)}$ associated to D9-branes (T-duality, κ-symmetry, susy algebra...).

The $A_{(10)}$ potential gauge and supersymmetry transformations must be consistent with those of the other fields[a]

Also its global ($SL(2,\mathbb{R})/SO(2) \sim SU(1,1)/U(1)$ S-duality) transformations must be consistent with those of the other fields.

[a] E.A. Bergshoeff, M. de Roo, B. Janssen and T. Ortín, *Nucl. Phys.* **B550** (1999) 289. `hep-th/9901055`. E.A. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *Class. Quant. Grav.* **18** (2001) 3359. `hep-th/0103233`.

[b] E.A. Bergshoeff, M. de Roo, S.F. Kerstan and F. Riccioni, *JHEP* **0508** (2005) 098. `hep-th/0506013`.

1 − $N = 2B, d = 10$ SUEGRA Revisited

It has long been known that $N = 2B, d = 10$ SUEGRA must contain a RR 10-form potential $A_{(10)}$ associated to D9-branes (T-duality, κ-symmetry, susy algebra...).

The $A_{(10)}$ potential gauge and supersymmetry transformations must be consistent with those of the other fields[a]

Also its global ($SL(2,\mathbb{R})/SO(2) \sim SU(1,1)/U(1)$ S-duality) transformations must be consistent with those of the other fields. But, what are they?

[a] E.A. Bergshoeff, M. de Roo, B. Janssen and T. Ortín, *Nucl. Phys.* **B550** (1999) 289. `hep-th/9901055`. E.A. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *Class. Quant. Grav.* **18** (2001) 3359. `hep-th/0103233`.

[b] E.A. Bergshoeff, M. de Roo, S.F. Kerstan and F. Riccioni, *JHEP* **0508** (2005) 098. `hep-th/0506013`.

1 − $N = 2B, d = 10$ SUEGRA Revisited

It has long been known that $N = 2B, d = 10$ SUEGRA must contain a RR 10-form potential $A_{(10)}$ associated to D9-branes (T-duality, κ-symmetry, susy algebra...).

The $A_{(10)}$ potential gauge and supersymmetry transformations must be consistent with those of the other fields[a]

Also its global ($SL(2,\mathbb{R})/SO(2) \sim SU(1,1)/U(1)$ S-duality) transformations must be consistent with those of the other fields. But, what are they?

$A_{(10)}$ cannot be a singlet under $SL(2,\mathbb{R})$. But finding exactly what it is requires an investigation of the behavior of **all** the fields of the theory under $SL(2,\mathbb{R})$[b].

[a] E.A. Bergshoeff, M. de Roo, B. Janssen and T. Ortín, *Nucl. Phys.* **B550** (1999) 289. hep-th/9901055. E.A. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *Class. Quant. Grav.* **18** (2001) 3359. hep-th/0103233.

[b] E.A. Bergshoeff, M. de Roo, S.F. Kerstan and F. Riccioni, *JHEP* **0508** (2005) 098. hep-th/0506013.

1 − $N = 2B, d = 10$ SUEGRA Revisited

It has long been known that $N = 2B, d = 10$ SUEGRA must contain a RR 10-form potential $A_{(10)}$ associated to D9-branes (T-duality, κ-symmetry, susy algebra...).

The $A_{(10)}$ potential gauge and supersymmetry transformations must be consistent with those of the other fields[a]

Also its global ($SL(2,\mathbb{R})/SO(2) \sim SU(1,1)/U(1)$ S-duality) transformations must be consistent with those of the other fields. But, what are they?

$A_{(10)}$ cannot be a singlet under $SL(2,\mathbb{R})$. But finding exactly what it is requires an investigation of the behavior of **all** the fields of the theory under $SL(2,\mathbb{R})$[b].

10-form potentials are special because they do not carry any continuous degree of freedom. Their existence has to be detected by imposing consistency (and non-triviality) of the susy algebra and gauge and $SL(2,\mathbb{R})$ transformations on the most general Ansatz.

---

[a] E.A. Bergshoeff, M. de Roo, B. Janssen and T. Ortín, *Nucl. Phys.* **B550** (1999) 289. hep-th/9901055. E.A. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *Class. Quant. Grav.* **18** (2001) 3359. hep-th/0103233.

[b] E.A. Bergshoeff, M. de Roo, S.F. Kerstan and F. Riccioni, *JHEP* **0508** (2005) 098. hep-th/0506013.

$N = 2B, d = 10$ SUEGRA

,

$N = 2B, d = 10$ SUEGRA

☞ Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.

,

$N = 2B, d = 10$ SUEGRA

- ☞ Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.
- ☞ Bosonic fields: $(\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

,

$N = 2B, d = 10$ SUEGRA

- ☞ Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.
- ☞ Bosonic fields: $(\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

$$\{e^a{}_\mu,$$

  - ➳ Zehnbein

$N = 2B, d = 10$ SUEGRA

- Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.
- Bosonic fields: ($\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

$$\{e_\mu{}^a, V_+^\alpha, V_-^\alpha,$$

  - Zehnbein
  - Scalars of $SU(1,1)/U(1)$

$N = 2B, d = 10$ SUEGRA

- Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.
- Bosonic fields: $(\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

$$\{e^a{}_\mu, V_+^\alpha, V_-^\alpha, A_{(2)}^\alpha,$$

  - Zehnbein
  - Scalars of $SU(1,1)/U(1)$
  - $SU(1,1)$ doublet of 2-forms

$N = 2B, d = 10$ SUEGRA

☞ Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.

☞ Bosonic fields: ($\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

$$\{e^a{}_\mu, V_+^\alpha, V_-^\alpha, A_{(2)}^\alpha, A_{(4)},$$

- Zehnbein
- Scalars of $SU(1,1)/U(1)$
- $SU(1,1)$ doublet of 2-forms
- S-duality-invariant 4-form

$N = 2B, d = 10$ SUEGRA

- Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.
- Bosonic fields: ($\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

$$\{e^a{}_\mu, V_+^\alpha, V_-^\alpha, A_{(2)}^\alpha, A_{(4)}, A_{(6)}^\alpha,$$

  - Zehnbein
  - Scalars of $SU(1,1)/U(1)$
  - $SU(1,1)$ doublet of 2-forms
  - S-duality-invariant 4-form
  - $SU(1,1)$ doublet of 6-forms

$N = 2B, d = 10$ SUEGRA

- Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.
- Bosonic fields: ($\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

$$\{e^a{}_\mu, V_+^\alpha, V_-^\alpha, A^\alpha_{(2)}, A_{(4)}, A^\alpha_{(6)}, A^{(\alpha\beta)}_{(8)},$$

  - Zehnbein
  - Scalars of $SU(1,1)/U(1)$
  - $SU(1,1)$ doublet of 2-forms
  - S-duality-invariant 4-form
  - $SU(1,1)$ doublet of 6-forms
  - $SU(1,1)$ triplet of 8-forms

$N = 2B, d = 10$ SUEGRA

- Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.
- Bosonic fields: ($\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

$$\{e^a{}_\mu, V_+^\alpha, V_-^\alpha, A_{(2)}^\alpha, A_{(4)}, A_{(6)}^\alpha, A_{(8)}^{(\alpha\beta)}, A_{(10)}^\alpha,$$

  - Zehnbein
  - Scalars of $SU(1,1)/U(1)$
  - $SU(1,1)$ doublet of 2-forms
  - S-duality-invariant 4-form
  - $SU(1,1)$ doublet of 6-forms
  - $SU(1,1)$ triplet of 8-forms
  - $SU(1,1)$ doublet of 10-forms

$N = 2B, d = 10$ SUEGRA

- Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.
- Bosonic fields: ($\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

$$\{e^a{}_\mu, V_+^\alpha, V_-^\alpha, A_{(2)}^\alpha, A_{(4)}, A_{(6)}^\alpha, A_{(8)}^{(\alpha\beta)}, A_{(10)}^\alpha, A_{(10)}^{(\alpha\beta\gamma)}\}$$

  - Zehnbein
  - Scalars of $SU(1,1)/U(1)$
  - $SU(1,1)$ doublet of 2-forms
  - S-duality-invariant 4-form
  - $SU(1,1)$ doublet of 6-forms
  - $SU(1,1)$ triplet of 8-forms
  - $SU(1,1)$ doublet of 10-forms
  - $SU(1,1)$ cuadruplet of 10-forms

$N = 2B, d = 10$ SUEGRA

☞ Fermionic fields: $\{\psi_\mu, \lambda\}$ complex, Majorana-Weyl spinors.

☞ Bosonic fields: ($\alpha, \beta = 1, 2$: $SU(1,1)$ indices. $+, -$: (local) $U(1)$ weights)

$$\{e^a{}_\mu, V_+^\alpha, V_-^\alpha, A^\alpha_{(2)}, A_{(4)}, A^\alpha_{(6)}, A^{(\alpha\beta)}_{(8)}, A^\alpha_{(10)}, A^{(\alpha\beta\gamma)}_{(10)}\}$$

- Zehnbein
- Scalars of $SU(1,1)/U(1)$
- $SU(1,1)$ doublet of 2-forms
- S-duality-invariant 4-form
- $SU(1,1)$ doublet of 6-forms
- $SU(1,1)$ triplet of 8-forms
- $SU(1,1)$ doublet of 10-forms
- $SU(1,1)$ cuadruplet of 10-forms

Which of these six 10-forms is the RR 10-form?

The RR 10-form has a characteristic dilaton factor in its supersymmetry transformation.

The RR 10-form has a characteristic dilaton factor in its supersymmetry transformation.

To find the dilaton factor we need to rewrite the scalars V_+^α, V_-^α in terms of the dilaton ϕ and RR 0-form $A_{(0)}$.

The RR 10-form has a characteristic dilaton factor in its supersymmetry transformation.

To find the dilaton factor we need to rewrite the scalars V_+^α, V_-^α in terms of the dilaton ϕ and RR 0-form $A_{(0)}$. This is done as follows:

The RR 10-form has a characteristic dilaton factor in its supersymmetry transformation.

To find the dilaton factor we need to rewrite the scalars V_+^α, V_-^α in terms of the dilaton ϕ and RR 0-form $A_{(0)}$. This is done as follows:

1. Fix the $U(1)$ gauge. Vgr. $V_+^\alpha \in \mathbb{R}$. Compensating $U(1)$ gauge transformations need to be introduced in supersymmetry transformations.

The RR 10-form has a characteristic dilaton factor in its supersymmetry transformation.

To find the dilaton factor we need to rewrite the scalars V_+^α, V_-^α in terms of the dilaton ϕ and RR 0-form $A_{(0)}$. This is done as follows:

1. Fix the $U(1)$ gauge. Vgr. $V_+^\alpha \in \mathbb{R}$. Compensating $U(1)$ gauge transformations need to be introduced in supersymmetry transformations.
2. Substitute the gauge-fixed variables $V_\pm^\alpha$ by the unconstrained $U(1)$-invariant $z = -V_-^\alpha / V_+^\alpha \in \mathbb{C}$ living in the unit disc and transforming fractional-linearly under $SU(1,1)$.

The RR 10-form has a characteristic dilaton factor in its supersymmetry transformation.

To find the dilaton factor we need to rewrite the scalars V_+^α, V_-^α in terms of the dilaton ϕ and RR 0-form $A_{(0)}$. This is done as follows:

1. Fix the $U(1)$ gauge. Vgr. $V_+^\alpha \in \mathbb{R}$. Compensating $U(1)$ gauge transformations need to be introduced in supersymmetry transformations.
2. Substitute the gauge-fixed variables $V_\pm^\alpha$ by the unconstrained $U(1)$-invariant $z = -V_-^\alpha / V_+^\alpha \in \mathbb{C}$ living in the unit disc and transforming fractional-linearly under $SU(1,1)$.
3. Change of variables from $z = \dfrac{1 + i\tau}{1 - i\tau}$ to τ living in the fundamental region of $SL(2,\mathbb{R})$.

The RR 10-form has a characteristic dilaton factor in its supersymmetry transformation.

To find the dilaton factor we need to rewrite the scalars V_+^α, V_-^α in terms of the dilaton ϕ and RR 0-form $A_{(0)}$. This is done as follows:

1. Fix the $U(1)$ gauge. Vgr. $V_+^\alpha \in \mathbb{R}$. Compensating $U(1)$ gauge transformations need to be introduced in supersymmetry transformations.
2. Substitute the gauge-fixed variables $V_\pm^\alpha$ by the unconstrained $U(1)$-invariant $z = -V_-^\alpha / V_+^\alpha \in \mathbb{C}$ living in the unit disc and transforming fractional-linearly under $SU(1,1)$.
3. Change of variables from $z = \dfrac{1 + i\tau}{1 - i\tau}$ to τ living in the fundamental region of $SL(2, \mathbb{R})$.

Conclusion: the RR 10-form belongs to the cuadruplet $A_{(10)}^{(\alpha\beta\gamma)}$

The RR 10-form has a characteristic dilaton factor in its supersymmetry transformation.

To find the dilaton factor we need to rewrite the scalars $V_+^{\alpha}, V_-^{\alpha}$ in terms of the dilaton ϕ and RR 0-form $A_{(0)}$. This is done as follows:

1. Fix the $U(1)$ gauge. Vgr. $V_+^{\alpha} \in \mathbb{R}$. Compensating $U(1)$ gauge transformations need to be introduced in supersymmetry transformations.
2. Substitute the gauge-fixed variables $V_{\pm}^{\alpha}$ by the unconstrained $U(1)$-invariant $z = -V_-^{\alpha}/V_+^{\alpha} \in \mathbb{C}$ living in the unit disc and transforming fractional-linearly under $SU(1,1)$.
3. Change of variables from $z = \dfrac{1+i\tau}{1-i\tau}$ to τ living in the fundamental region of $SL(2,\mathbb{R})$.

Conclusion: the RR 10-form belongs to the cuadruplet $A_{(10)}^{(\alpha\beta\gamma)}$

What are the S-duals of the D9-branes?

2 – Potentials and Branes

p-branes couple naturally to $(p+1)$-form potentials

2 – Potentials and Branes

p-branes couple naturally to $(p+1)$-form potentials

The coupling is completely determined by κ-symmetry which requires (gauge-fixed) efective actions of the general form

$$\mathcal{L}_{\text{brane}} = \tau_{\text{brane}} \sqrt{|g|} + \epsilon^{\mu_1 \cdots \mu_{p+1}} A_{(p+1)\mu_1 \cdots \mu_{p+1}} ,$$

and a precise relation between τ_{brane} (a function of scalars) and $A_{(p+1)\mu_1 \cdots \mu_{p+1}}$.

2 – Potentials and Branes

p-branes couple naturally to $(p+1)$-form potentials

The coupling is completely determined by κ-symmetry which requires (gauge-fixed) efective actions of the general form

$$\mathcal{L}_{\text{brane}} = \tau_{\text{brane}} \sqrt{|g|} + \epsilon^{\mu_1 \cdots \mu_{p+1}} A_{(p+1)\mu_1 \cdots \mu_{p+1}} \, ,$$

and a precise relation between τ_{brane} (a function of scalars) and $A_{(p+1)\mu_1 \cdots \mu_{p+1}}$.

That relation is dictated by invariance under supersymmetry that we can chek at leading order in gravitinos.

2 – Potentials and Branes

p-branes couple naturally to $(p+1)$-form potentials

The coupling is completely determined by κ-symmetry which requires (gauge-fixed) efective actions of the general form

$$\mathcal{L}_{\text{brane}} = \tau_{\text{brane}} \sqrt{|g|} + \epsilon^{\mu_1 \cdots \mu_{p+1}} A_{(p+1)\mu_1 \cdots \mu_{p+1}} \,,$$

and a precise relation between τ_{brane} (a function of scalars) and $A_{(p+1)\mu_1 \cdots \mu_{p+1}}$.

That relation is dictated by invariance under supersymmetry that we can chek at leading order in gravitinos. The relevant transformations are

$$\delta_\epsilon g_{\mu\nu} = 2i\bar{\epsilon}\gamma_{(\mu}\psi_{\nu)} + \text{h.c.}\,, \qquad \delta_\epsilon A_{\mu_1 \cdots \mu_{p+1}} \sim f\, \bar{\epsilon}\gamma_{[\mu_1 \cdots \mu_p}\sigma\psi_{\mu_{p+1}]}\,,$$

where f is a function of scalars σ a theory-dependent matrix.

2 – Potentials and Branes

p-branes couple naturally to $(p+1)$-form potentials

The coupling is completely determined by κ-symmetry which requires (gauge-fixed) efective actions of the general form

$$\mathcal{L}_{\text{brane}} = \tau_{\text{brane}} \sqrt{|g|} + \epsilon^{\mu_1 \cdots \mu_{p+1}} A_{(p+1)\mu_1 \cdots \mu_{p+1}} ,$$

and a precise relation between τ_{brane} (a function of scalars) and $A_{(p+1)\mu_1 \cdots \mu_{p+1}}$.

That relation is dictated by invariance under supersymmetry that we can chek at leading order in gravitinos. The relevant transformations are

$$\delta_\epsilon g_{\mu\nu} = 2i\bar{\epsilon}\gamma_{(\mu}\psi_{\nu)} + \text{h.c.} , \qquad \delta_\epsilon A_{\mu_1 \cdots \mu_{p+1}} \sim f\, \bar{\epsilon}\gamma_{[\mu_1 \cdots \mu_p}\sigma\psi_{\mu_{p+1}]} ,$$

where f is a function of scalars σ a theory-dependent matrix. In general we find

$$\delta_\epsilon \mathcal{L}_{\text{brane}} \sim \bar{\psi}_\mu \gamma^\mu \left(\tau_{\text{brane}} 1 + f \gamma_{01 \cdots p}\, \sigma\right)\epsilon .$$

2 – Potentials and Branes

p-branes couple naturally to $(p+1)$-form potentials

The coupling is completely determined by κ-symmetry which requires (gauge-fixed) efective actions of the general form

$$\mathcal{L}_{\rm brane} = \tau_{\rm brane}\sqrt{|g|} + \epsilon^{\mu_1\cdots\mu_{p+1}} A_{(p+1)\mu_1\cdots\mu_{p+1}}\,,$$

and a precise relation between $\tau_{\rm brane}$ (a function of scalars) and $A_{(p+1)\mu_1\cdots\mu_{p+1}}$.

That relation is dictated by invariance under supersymmetry that we can chek at leading order in gravitinos. The relevant transformations are

$$\delta_\epsilon g_{\mu\nu} = 2i\bar{\epsilon}\gamma_{(\mu}\psi_{\nu)} + \text{h.c.}\,, \qquad \delta_\epsilon A_{\mu_1\cdots\mu_{p+1}} \sim f\,\bar{\epsilon}\gamma_{[\mu_1\cdots\mu_p}\sigma\psi_{\mu_{p+1}]}\,,$$

where f is a function of scalars σ a theory-dependent matrix. In general we find

$$\delta_\epsilon\mathcal{L}_{\rm brane} \sim \bar{\psi}_\mu\gamma^\mu\left(\tau_{\rm brane}1 + f\gamma_{01\cdots p}\,\sigma\right)\epsilon\,.$$

This variation is proportional to the projection operator that annihilates ϵ iff

$$\tau_{\rm brane} = f\,,$$

which determines the brane tension.

3 – IIB Strings

Example: Let us consider the IIB objects that couple to the doublet of 2-forms $A^{\alpha}_{(2)} = (C_{\mu\nu}, B_{\mu\nu})$ (D1 and F1) whose supersymmetry transformations are

$$\delta_\epsilon C_{\mu\nu} = -8ie^{-\phi}\bar{\epsilon}\sigma_1\gamma_{[\mu}\psi_{\nu]} + \ell\delta_\epsilon B_{\mu\nu} . \quad \delta_\epsilon B_{\mu\nu} = 8i\bar{\epsilon}\sigma_3\gamma_{[\mu}\psi_{\nu]} ,$$

3 – IIB Strings

Example: Let us consider the IIB objects that couple to the doublet of 2-forms $A^{\alpha}_{(2)} = (C_{\mu\nu}, B_{\mu\nu})$ (D1 and F1) whose supersymmetry transformations are

$$\delta_\epsilon C_{\mu\nu} = -8ie^{-\phi}\bar{\epsilon}\sigma_1\gamma_{[\mu}\psi_{\nu]} + \ell\delta_\epsilon B_{\mu\nu} \,. \qquad \delta_\epsilon B_{\mu\nu} = 8i\bar{\epsilon}\,\sigma_3\gamma_{[\mu}\psi_{\nu]} \,,$$

Integrating out the Born-Infeld field their effective actions are

$$\mathcal{L}_{\rm D1} = \tau_{\rm D1}\sqrt{|g|} + \tfrac{1}{4}\epsilon^{\mu\nu}C_{\mu\nu} \,, \qquad\qquad \mathcal{L}_{\rm F1} = \tau_{\rm F1}\sqrt{|g|} + \tfrac{1}{4}\epsilon^{\mu\nu}B_{\mu\nu} \,.$$

3 – IIB Strings

Example: Let us consider the IIB objects that couple to the doublet of 2-forms $A^{\alpha}_{(2)} = (C_{\mu\nu}, B_{\mu\nu})$ (D1 and F1) whose supersymmetry transformations are

$$\delta_{\epsilon} C_{\mu\nu} = -8ie^{-\phi}\bar{\epsilon}\sigma_1\gamma_{[\mu}\psi_{\nu]} + \ell\delta_{\epsilon}B_{\mu\nu}\,. \qquad \delta_{\epsilon}B_{\mu\nu} = 8i\bar{\epsilon}\,\sigma_3\gamma_{[\mu}\psi_{\nu]}\,,$$

Integrating out the Born-Infeld field their effective actions are

$$\mathcal{L}_{\rm D1} = \tau_{\rm D1}\sqrt{|g|} + \tfrac{1}{4}\epsilon^{\mu\nu}C_{\mu\nu}\,, \qquad \mathcal{L}_{\rm F1} = \tau_{\rm F1}\sqrt{|g|} + \tfrac{1}{4}\epsilon^{\mu\nu}B_{\mu\nu}\,.$$

The supersymmetry variation of the actions are

$$\delta_{\epsilon}\mathcal{L}_{\rm F1} \sim (\bar{\psi}_{\mu}\gamma^{\mu})\tfrac{1}{2}\left(\tau_{\rm F1}1 + \sigma_3\gamma_{01}\right)\epsilon \qquad \Rightarrow \qquad \tau_{\rm F1} = 1\,.$$

$$\delta_{\epsilon}\mathcal{L}_{\rm D1} \sim (\bar{\psi}_{\mu}\gamma^{\mu})\tfrac{1}{2}\left[\tau_{\rm D1}1 - (e^{-\phi}\sigma_1 - \ell\sigma_3)\gamma_{01}\right]\epsilon \qquad \Rightarrow \qquad \tau_{\rm D1} = \sqrt{e^{-2\phi} + \ell^2}\,.$$

3 – IIB Strings

Example: Let us consider the IIB objects that couple to the doublet of 2-forms $A^{\alpha}_{(2)} = (C_{\mu\nu}, B_{\mu\nu})$ (D1 and F1) whose supersymmetry transformations are

$$\delta_\epsilon C_{\mu\nu} = -8ie^{-\phi}\bar{\epsilon}\sigma_1\gamma_{[\mu}\psi_{\nu]} + \ell\delta_\epsilon B_{\mu\nu}\,. \qquad \delta_\epsilon B_{\mu\nu} = 8i\bar{\epsilon}\,\sigma_3\gamma_{[\mu}\psi_{\nu]}\,,$$

Integrating out the Born-Infeld field their effective actions are

$$\mathcal{L}_{\rm D1} = \tau_{\rm D1}\sqrt{|g|} + \tfrac{1}{4}\epsilon^{\mu\nu}C_{\mu\nu}\,, \qquad \mathcal{L}_{\rm F1} = \tau_{\rm F1}\sqrt{|g|} + \tfrac{1}{4}\epsilon^{\mu\nu}B_{\mu\nu}\,.$$

The supersymmetry variation of the actions are

$$\delta_\epsilon\mathcal{L}_{\rm F1} \sim (\bar{\psi}_\mu\gamma^\mu)\tfrac{1}{2}\left(\tau_{\rm F1}1 + \sigma_3\gamma_{01}\right)\epsilon \qquad \Rightarrow \qquad \tau_{\rm F1} = 1\,.$$

$$\delta_\epsilon\mathcal{L}_{\rm D1} \sim (\bar{\psi}_\mu\gamma^\mu)\tfrac{1}{2}\left[\tau_{\rm D1}1 - (e^{-\phi}\sigma_1 - \ell\sigma_3)\gamma_{01}\right]\epsilon \qquad \Rightarrow \qquad \tau_{\rm D1} = \sqrt{e^{-2\phi} + \ell^2}\,.$$

For (p,q)-strings

$$\mathcal{L}_{\rm (p,q)} = \tau_{(p,q)}\sqrt{|g|} + \tfrac{1}{4}\epsilon^{\mu\nu}\left(pB_{\mu\nu} + q\,C_{\mu\nu}\right),$$

$$\delta\mathcal{L}_{(p,q)} \sim \left(\tau_{(p,q)}1 + ((p+\ell q)\sigma_3 - e^{-\phi} q\sigma_1)\gamma_{01}\right)\epsilon\,. \quad \Rightarrow \quad \tau_{p,q} = \sqrt{(p+\ell q)^2 + e^{-2\phi}q^2}\,,$$

$$\delta\mathcal{L}_{(p,q)} \sim \Big(\tau_{(p,q)}1 + ((p+\ell q)\sigma_3 - e^{-\phi}q\sigma_1)\gamma_{01}\Big)\,\epsilon\,. \quad \Rightarrow \quad \tau_{p,q} = \sqrt{(p+\ell q)^2 + e^{-2\phi}q^2}\,,$$

In the Einstein frame it can be written in the manifestly $SL(2,\mathbb{R})$-invariant form

$$\tau^{\mathrm{E}}_{(p,q)} = \sqrt{q^\alpha q^\beta \mathcal{M}_{\alpha\beta}}\,, \qquad (q^\alpha) = \begin{pmatrix} q \\ p \end{pmatrix}\,, \qquad (\mathcal{M}_{\alpha\beta}) = e^{+\phi}\begin{pmatrix} |\tau|^2 & \ell \\ \ell & 1 \end{pmatrix}$$

$$\delta\mathcal{L}_{(p,q)} \sim \left(\tau_{(p,q)}1 + ((p+\ell q)\sigma_3 - e^{-\phi}q\sigma_1)\gamma_{01}\right)\epsilon\,. \quad \Rightarrow \quad \tau_{p,q} = \sqrt{(p+\ell q)^2 + e^{-2\phi}q^2}\,,$$

In the Einstein frame it can be written in the manifestly $SL(2,\mathbb{R})$-invariant form

$$\tau^{\mathrm{E}}_{(p,q)} = \sqrt{q^\alpha q^\beta \mathcal{M}_{\alpha\beta}}\,, \qquad (q^\alpha) = \begin{pmatrix} q \\ p \end{pmatrix}\,, \qquad (\mathcal{M}_{\alpha\beta}) = e^{+\phi}\begin{pmatrix} |\tau|^2 & \ell \\ \ell & 1 \end{pmatrix}$$

Summary of results of $p < 7$-branes:

potential	brane	tension	projection operator
$C_{(2)}$	D1	$\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1 + \frac{-e^{-\phi}\sigma_1+\ell\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\gamma_{01}\right)$
$B_{(2)}$	F1	1	$\frac{1}{2}\left(1+\sigma_3\gamma_{01}\right)$
$C_{(4)}$	D3	$e^{-\phi}$	$\frac{1}{2}(1+i\sigma_2\gamma_{0123})$
$C_{(6)}$	D5	$e^{-\phi}$	$\frac{1}{2}(1+\sigma_1\gamma_{01\cdots5})$
$B_{(6)}$	NS5	$e^{-\phi}\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1 + \frac{e^{-\phi}\sigma_3+\ell\sigma_1}{\sqrt{e^{-2\phi}+\ell^2}}\gamma_{01\cdots5}\right)$

4 – IIB 7-Branes

7-branes provide a less trivial example.

4 – IIB 7-Branes

7-branes provide a less trivial example.

There is a triplet of 8-forms $A_{(8)}^{(\alpha\beta)} \equiv (C_{(8)}, D_{(8)}, B_{(8)})$ and they correspond to the D7, I7 and $\widetilde{\text{D7}}$ resp.

4 – IIB 7-Branes

7-branes provide a less trivial example.

There is a triplet of 8-forms $A_{(8)}^{(\alpha\beta)} \equiv (C_{(8)}, D_{(8)}, B_{(8)})$ and they correspond to the D7, I7 and $\widetilde{\text{D7}}$ resp.

$\widetilde{\text{D7}}$ and D7 are S-duals (under the $\mathbb{Z}_2$ $\tau \rightarrow -1/\tau$).

4 – IIB 7-Branes

7-branes provide a less trivial example.

There is a triplet of 8-forms $A_{(8)}^{(\alpha\beta)} \equiv (C_{(8)}, D_{(8)}, B_{(8)})$ and they correspond to the D7, I7 and $\widetilde{\text{D7}}$ resp.

$\widetilde{\text{D7}}$ and D7 are S-duals (under the $\mathbb{Z}_2$ $\tau \to -1/\tau$).

I7 is invariant under S-duality.

4 – IIB 7-Branes

7-branes provide a less trivial example.

There is a triplet of 8-forms $A_{(8)}^{(\alpha\beta)} \equiv (C_{(8)}, D_{(8)}, B_{(8)})$ and they correspond to the D7, I7 and $\widetilde{\text{D7}}$ resp.

$\widetilde{\text{D7}}$ and D7 are S-duals (under the $\mathbb{Z}_2$ $\tau \to -1/\tau$).

I7 is invariant under S-duality.

Following the same procedure for each separate kind of 7-brane one gets

potential	brane	tension	projection operator
$C_{(8)}$	D7	$e^{-\phi}$	$\frac{1}{2}(1 + i\gamma_{01\cdots 7}\sigma_2)$
$D_{(8)}$	I7	$\ell e^{-\phi}$	$\frac{1}{2}(1 + i\gamma_{01\cdots 7}\sigma_2)$
$B_{(8)}$	$\widetilde{\text{D7}}$	$e^{-\phi}(e^{-2\phi} + \ell^2)$	$\frac{1}{2}(1 + i\gamma_{01\cdots 7}\sigma_2)$

Consider now the action of a combination of 7-branes:

$$\mathcal{L}_{(p,r,q)} \sim \tau_{(p,r,q)} \sqrt{|g|} + \epsilon^{\mu_1 \cdots \mu_8} \left(pC_{\mu_1 \cdots \mu_8} + rD_{\mu_1 \cdots \mu_8} + qB_{\mu_1 \cdots \mu_8} \right) .$$

$$\begin{array}{rcl} \text{D7-brane} & \longrightarrow & (p,r,q) = (1,0,0) \\ \widetilde{\text{D7}}\text{-brane} & \longrightarrow & (p,r,q) = (0,0,1) \\ \text{I7-brane} & \longrightarrow & (p,r,q) = (0,1,0) \end{array}$$

Consider now the action of a combination of 7-branes:

$$\mathcal{L}_{(p,r,q)} \sim \tau_{(p,r,q)} \sqrt{|g|} + \epsilon^{\mu_1 \cdots \mu_8} \left(pC_{\mu_1 \cdots \mu_8} + rD_{\mu_1 \cdots \mu_8} + qB_{\mu_1 \cdots \mu_8} \right) .$$

$$\begin{array}{rcl} \text{D7-brane} & \longrightarrow & (p,r,q) = (1,0,0) \\ \widetilde{\text{D7}}\text{-brane} & \longrightarrow & (p,r,q) = (0,0,1) \\ \text{I7-brane} & \longrightarrow & (p,r,q) = (0,1,0) \end{array}$$

One finds to leading order in the gravitino

$$\delta_\epsilon \mathcal{L}_{(p,r,q)} \sim \bar{\psi}_\mu \gamma^\mu \left[\tau_{(p,r,q)} 1 + i \big(p\, e^{-\phi} + r\, \ell e^{-\phi} + q\, e^{-\phi} (e^{-2\phi} + \ell^2) \big) \gamma_{01 \cdots 7} \sigma_2 \right] \epsilon \, .$$

which is proportional to a projection operator provided that

$$\tau_{(p,r,q)} = e^{-\phi} |p + r\, \ell + q\, (e^{-2\phi} + \ell^2)| \, .$$

Consider now the action of a combination of 7-branes:

$$\mathcal{L}_{(p,r,q)} \sim \tau_{(p,r,q)} \sqrt{|g|} + \epsilon^{\mu_1 \cdots \mu_8} \big(p C_{\mu_1 \cdots \mu_8} + r D_{\mu_1 \cdots \mu_8} + q B_{\mu_1 \cdots \mu_8} \big) \, .$$

$$\begin{array}{rcl} \text{D7-brane} & \longrightarrow & (p,r,q) = (1,0,0) \\ \widetilde{\text{D7}}\text{-brane} & \longrightarrow & (p,r,q) = (0,0,1) \\ \text{I7-brane} & \longrightarrow & (p,r,q) = (0,1,0) \end{array}$$

One finds to leading order in the gravitino

$$\delta_\epsilon \mathcal{L}_{(p,r,q)} \sim \bar{\psi}_\mu \gamma^\mu \big[\tau_{(p,r,q)} 1 + i \big(p\, e^{-\phi} + r\, \ell e^{-\phi} + q\, e^{-\phi} (e^{-2\phi} + \ell^2) \big) \gamma_{01\cdots 7} \sigma_2 \big] \epsilon \, .$$

which is proportional to a projection operator provided that

$$\tau_{(p,r,q)} = e^{-\phi} | p + r\, \ell + q\, (e^{-2\phi} + \ell^2) | \, .$$

In the Einstein frame, this tension formula can be written in manifest $SL(2,\mathbb{R})$-invariant form:

$$\tau^{\mathrm{E}}_{(p,r,q)} = | q^{\alpha\beta} \mathcal{M}_{\alpha\beta} | \, , \qquad (q^{\alpha\beta}) = \begin{pmatrix} q & r/2 \\ r/2 & p \end{pmatrix} ,$$

The determinant of the charge matrix $q^{\alpha\beta}$ is S-duality-invariant

$$\det\left[q^{\alpha\beta}\right] = pq - \frac{r^2}{4} \equiv -\alpha^2 \,,$$

for some α. These are separate orbits of S-duality.

The determinant of the charge matrix $q^{\alpha\beta}$ is S-duality-invariant

$$\det\left[q^{\alpha\beta}\right] = pq - \frac{r^2}{4} \equiv -\alpha^2\,,$$

for some α. These are separate orbits of S-duality.

$\Rightarrow$ 7-branes fall into conjugacy classes characterized by the value of α.

The determinant of the charge matrix $q^{\alpha\beta}$ is S-duality-invariant

$$\det\left[q^{\alpha\beta}\right] = pq - \frac{r^2}{4} \equiv -\alpha^2\,,$$

for some α. These are separate orbits of S-duality.

$\Rightarrow$ 7-branes fall into conjugacy classes characterized by the value of α. Each conjugacy class is a 2-dimensional non-linear representation of $SL(2,\mathbb{R})$ that can be obtained by solving r in terms of p and q:

$$r(p,q) = \pm 2\sqrt{pq + \alpha^2}\,.$$

The determinant of the charge matrix $q^{\alpha\beta}$ is S-duality-invariant

$$\det\left[q^{\alpha\beta}\right] = pq - \frac{r^2}{4} \equiv -\alpha^2 \, ,$$

for some α. These are separate orbits of S-duality.

$\Rightarrow$ 7-branes fall into conjugacy classes characterized by the value of α. Each conjugacy class is a 2-dimensional non-linear representation of $SL(2,\mathbb{R})$ that can be obtained by solving r in terms of p and q:

$$r(p,q) = \pm 2\sqrt{pq + \alpha^2} \, .$$

The elements of these representations form 2-dimensional manifolds which are homogenous spaces $SL(2,\mathbb{R})/H_\alpha$ where H_α is the isotropy subgrgroup of the α conjugacy class.

The determinant of the charge matrix $q^{\alpha\beta}$ is S-duality-invariant

$$\det\left[q^{\alpha\beta}\right] = pq - \frac{r^2}{4} \equiv -\alpha^2\,,$$

for some α. These are separate orbits of S-duality.

$\Rightarrow$ 7-branes fall into conjugacy classes characterized by the value of α. Each conjugacy class is a 2-dimensional non-linear representation of $SL(2,\mathbb{R})$ that can be obtained by solving r in terms of p and q:

$$r(p,q) = \pm 2\sqrt{pq+\alpha^2}\,.$$

The elements of these representations form 2-dimensional manifolds which are homogenous spaces $SL(2,\mathbb{R})/H_\alpha$ where H_α is the isotropy subrgroup of the α conjugacy class. D7- and $\widetilde{\mathrm{D7}}$-branes belong to the $\alpha = 0$ conjugacy class.

I7-branes belong to $\alpha^2 > 0$ conjugacy classes.

5 – IIB 9-Branes

We finally arrive to 9-brane case.

5 – IIB 9-Branes

We finally arrive to 9-brane case.

There is a doublet and a quadruplet of 10-forms.

5 – IIB 9-Branes

We finally arrive to 9-brane case.

There is a doublet and a quadruplet of 10-forms.

For the doublet of 10-forms supersymmetry leads to

potential	brane	tension	projection operator
$\mathcal{D}_{(10)}$	S9	$e^{-2\phi}$	$\frac{1}{2}(1+\sigma_3)$
$\mathcal{E}_{(10)}$	$\widetilde{\text{S9}}$	$e^{-2\phi}\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1+\frac{-e^{-\phi}\sigma_1+\ell\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\right)$

5 – IIB 9-Branes

We finally arrive to 9-brane case.

There is a doublet and a quadruplet of 10-forms.

For the doublet of 10-forms supersymmetry leads to

potential	brane	tension	projection operator
$\mathcal{D}_{(10)}$	S9	$e^{-2\phi}$	$\frac{1}{2}(1+\sigma_3)$
$\mathcal{E}_{(10)}$	$\widetilde{\text{S9}}$	$e^{-2\phi}\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1+\frac{-e^{-\phi}\sigma_1+\ell\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\right)$

The tension of a (p,q)-9-brane is given by

$$\tau_{(p,q)} = e^{-2\phi}\sqrt{(p+\ell q)^2 + e^{-2\phi}q^2}\,.$$

In Einstein frame the tension is again

$$\tau^{\mathrm{E}}_{(p,q)} = \sqrt{q^\alpha q^\beta \mathcal{M}_{\alpha\beta}}\,, \qquad (q^\alpha) = \begin{pmatrix} q \\ p \end{pmatrix}, \qquad (\mathcal{M}_{\alpha\beta}) = e^{+\phi}\begin{pmatrix} |\tau|^2 & \ell \\ \ell & 1 \end{pmatrix}$$

For the quadruplet we find

potential	brane	charge	tension τ and projection operator P
$C_{(10)}$	D9	q	$\tau = e^{-\phi}$ $P = \frac{1}{2}(1+\sigma_1)$
$D_{(10)}$	–	r	$\tau = e^{-\phi}\sqrt{\frac{1}{9}\,e^{-2\phi} + \ell^2}$ $P = \frac{1}{2}\left(1 + \frac{\ell\sigma_1 + \frac{1}{3}e^{-\phi}\sigma_3}{\sqrt{\frac{1}{9}e^{-2\phi}+\ell^2}}\right)$
$E_{(10)}$	–	s	$\tau = e^{-\phi}\sqrt{(\frac{1}{3}e^{-2\phi} + \ell^2)^2 + \frac{4}{9}\,\ell^2 e^{-2\phi}}$ $P = \frac{1}{2}\left(1 - \frac{(\frac{1}{3}\,e^{-2\phi}+\ell^2)\sigma_1 + \frac{2}{3}\ell e^{-\phi}\sigma_3}{\sqrt{(\frac{1}{3}e^{-2\phi}+\ell^2)^2+\frac{4}{9}\ell^2 e^{-2\phi}}}\right)$
$B_{(10)}$	$\widetilde{\mathrm{D9}}$	p	$\tau = e^{-\phi}\left(e^{-2\phi} + \ell^2\right)^{3/2}$ $P = \frac{1}{2}\left(1 - \frac{\ell\sigma_1 + e^{-\phi}\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\right)$

For the quadruplet we find

potential	brane	charge	tension τ and projection operator P
$C_{(10)}$	D9	q	$\tau = e^{-\phi}$ $P = \frac{1}{2}(1+\sigma_1)$
$D_{(10)}$	–	r	$\tau = e^{-\phi}\sqrt{\frac{1}{9}\,e^{-2\phi}+\ell^2}$ $P = \frac{1}{2}\left(1+\frac{\ell\sigma_1+\frac{1}{3}e^{-\phi}\sigma_3}{\sqrt{\frac{1}{9}e^{-2\phi}+\ell^2}}\right)$
$E_{(10)}$	–	s	$\tau = e^{-\phi}\sqrt{(\frac{1}{3}e^{-2\phi}+\ell^2)^2+\frac{4}{9}\,\ell^2 e^{-2\phi}}$ $P = \frac{1}{2}\left(1-\frac{(\frac{1}{3}\,e^{-2\phi}+\ell^2)\sigma_1+\frac{2}{3}\ell e^{-\phi}\sigma_3}{\sqrt{(\frac{1}{3}e^{-2\phi}+\ell^2)^2+\frac{4}{9}\ell^2 e^{-2\phi}}}\right)$
$B_{(10)}$	$\widetilde{\text{D9}}$	p	$\tau = e^{-\phi}\left(e^{-2\phi}+\ell^2\right)^{3/2}$ $P = \frac{1}{2}\left(1-\frac{\ell\sigma_1+e^{-\phi}\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\right)$

The tension of a (p,r,s,q)-brane is given by

$$\begin{aligned}\tau_{(p,r,s,q)} \quad = \quad & \{[e^{-\phi}p+\ell e^{-\phi}r-\left(\tfrac{1}{3}e^{-3\phi}+\ell^2 e^{-\phi}\right)s-\left(\ell^3 e^{-\phi}+\ell e^{-3\phi}\right)q]^2\\ & +[\tfrac{1}{3}e^{-2\phi}r-\tfrac{2}{3}\ell e^{-2\phi}s-\left(e^{-4\phi}+\ell^2 e^{-2\phi}\right)q]^2\}^{1/2}\,.\end{aligned}$$

In Einstein frame the manifest $SL(2,\mathbb{R})$-invariant tension is given by

$$\tau^{\mathrm{E}}_{(p,r,s,q)} = \sqrt{q^{\alpha\beta\gamma} q^{\delta\epsilon\zeta} \mathcal{M}_{\alpha\beta} \mathcal{M}_{\delta\epsilon} \mathcal{M}_{\gamma\zeta}}\,,$$

where

$$q^{222} \equiv p\,, \quad q^{122} \equiv -r/3\,, \quad q^{112} \equiv -s/3\,, \quad q^{111} \equiv q\,.$$

In Einstein frame the manifest $SL(2,\mathbb{R})$-invariant tension is given by

$$\tau^{\mathrm{E}}_{(p,r,s,q)} = \sqrt{q^{\alpha\beta\gamma} q^{\delta\epsilon\zeta} \mathcal{M}_{\alpha\beta} \mathcal{M}_{\delta\epsilon} \mathcal{M}_{\gamma\zeta}}\,,$$

where

$$q^{222} \equiv p\,, \quad q^{122} \equiv -r/3\,, \quad q^{112} \equiv -s/3\,, \quad q^{111} \equiv q\,.$$

In the previous cases the cancellation of the gravitino terms in the supersymmetry variations did not lead to new conditions.

In Einstein frame the manifest $SL(2,\mathbb{R})$-invariant tension is given by

$$\tau^{\rm E}_{(p,r,s,q)} = \sqrt{q^{\alpha\beta\gamma} q^{\delta\epsilon\zeta} \mathcal{M}_{\alpha\beta} \mathcal{M}_{\delta\epsilon} \mathcal{M}_{\gamma\zeta}}\,,$$

where

$$q^{222} \equiv p\,, \quad q^{122} \equiv -r/3\,, \quad q^{112} \equiv -s/3\,, \quad q^{111} \equiv q\,.$$

In the previous cases the cancellation of the gravitino terms in the supersymmetry variations did not lead to new conditions.

For (p,r,s,q)-branes the dilatino terms only cancel iff

$$3qr + s^2 = 0\,, \qquad 3ps + r^2 = 0\,, \qquad 9pq - rs = 0\,.$$

In Einstein frame the manifest $SL(2,\mathbb{R})$-invariant tension is given by

$$\tau^{\mathrm{E}}_{(p,r,s,q)} = \sqrt{q^{\alpha\beta\gamma}q^{\delta\epsilon\zeta}\mathcal{M}_{\alpha\beta}\mathcal{M}_{\delta\epsilon}\mathcal{M}_{\gamma\zeta}}\,,$$

where

$$q^{222} \equiv p\,, \quad q^{122} \equiv -r/3\,, \quad q^{112} \equiv -s/3\,, \quad q^{111} \equiv q\,.$$

In the previous cases the cancellation of the gravitino terms in the supersymmetry variations did not lead to new conditions.

For (p,r,s,q)-branes the dilatino terms only cancel iff

$$3qr + s^2 = 0\,, \qquad 3ps + r^2 = 0\,, \qquad 9pq - rs = 0\,.$$

The D9-brane and the $\widetilde{\mathrm{D9}}$-brane satisfy them, but not the other 9-branes!.

In Einstein frame the manifest $SL(2,\mathbb{R})$-invariant tension is given by

$$\tau^{\rm E}_{(p,r,s,q)} = \sqrt{q^{\alpha\beta\gamma}q^{\delta\epsilon\zeta}\mathcal{M}_{\alpha\beta}\mathcal{M}_{\delta\epsilon}\mathcal{M}_{\gamma\zeta}}\,,$$

where

$$q^{222} \equiv p\,, \quad q^{122} \equiv -r/3\,, \quad q^{112} \equiv -s/3\,, \quad q^{111} \equiv q\,.$$

In the previous cases the cancellation of the gravitino terms in the supersymmetry variations did not lead to new conditions.

For (p,r,s,q)-branes the dilatino terms only cancel iff

$$3qr + s^2 = 0\,, \qquad 3ps + r^2 = 0\,, \qquad 9pq - rs = 0\,.$$

The D9-brane and the $\widetilde{\rm D9}$-brane satisfy them, but not the other 9-branes!.

Let's introduce

$$Q^{\alpha\beta} \equiv q^{\alpha\gamma\delta}q^{\beta\epsilon\zeta}\epsilon_{\gamma\epsilon}\epsilon_{\delta\zeta} = \tfrac{1}{9}\begin{pmatrix} 2(3qr+s^2) & 9pq-rs \\ 9pq-rs & 2(3ps+r^2) \end{pmatrix}.$$

In Einstein frame the manifest $SL(2,\mathbb{R})$-invariant tension is given by

$$\tau^{\mathrm{E}}_{(p,r,s,q)} = \sqrt{q^{\alpha\beta\gamma} q^{\delta\epsilon\zeta} \mathcal{M}_{\alpha\beta}\mathcal{M}_{\delta\epsilon}\mathcal{M}_{\gamma\zeta}}\,,$$

where

$$q^{222} \equiv p\,, \quad q^{122} \equiv -r/3\,, \quad q^{112} \equiv -s/3\,, \quad q^{111} \equiv q\,.$$

In the previous cases the cancellation of the gravitino terms in the supersymmetry variations did not lead to new conditions.

For (p,r,s,q)-branes the dilatino terms only cancel iff

$$3qr + s^2 = 0\,, \qquad 3ps + r^2 = 0\,, \qquad 9pq - rs = 0\,.$$

The D9-brane and the $\widetilde{\mathrm{D9}}$-brane satisfy them, but not the other 9-branes!.

Let's introduce

$$Q^{\alpha\beta} \equiv q^{\alpha\gamma\delta} q^{\beta\epsilon\zeta} \epsilon_{\gamma\epsilon}\epsilon_{\delta\zeta} = \tfrac{1}{9} \begin{pmatrix} 2(3qr+s^2) & 9pq - rs \\ 9pq - rs & 2(3ps + r^2) \end{pmatrix}.$$

The supersymmetry constraints are just the triplet $Q^{\alpha\beta} = 0$.

They can be used to solve for r, s in terms of p, q and we end up with a set of (p,q) 9-branes that define a two-dimensional manifold in a four-dimensional space. Intrinsically, it is the same homogenous space as in the D7-brane case.

There are no other conjugacy classes of half-supersymmetric 9-branes.

There are no other conjugacy classes of half-supersymmetric 9-branes.

Let us assume the 9-brane charges are quantized. The S-duality group becomes $SL(2,\mathbb{Z})$.

There are no other conjugacy classes of half-supersymmetric 9-branes.

Let us assume the 9-brane charges are quantized. The S-duality group becomes $SL(2,\mathbb{Z})$.

The $SL(2,\mathbb{Z})$ orbit of the S9-brane of the linear doublet is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix} .$$

There are no other conjugacy classes of half-supersymmetric 9-branes.

Let us assume the 9-brane charges are quantized. The S-duality group becomes $SL(2,\mathbb{Z})$.

The $SL(2,\mathbb{Z})$ orbit of the S9-brane of the linear doublet is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix} .$$

It can be shown that or any pair a, c of co-prime integers there exist integers b and d $ad - bc = 1$ and so all branes are in the $SL(2,\mathbb{Z})$ orbit of the S9-brane.

There are no other conjugacy classes of half-supersymmetric 9-branes.

Let us assume the 9-brane charges are quantized. The S-duality group becomes $SL(2,\mathbb{Z})$.

The $SL(2,\mathbb{Z})$ orbit of the S9-brane of the linear doublet is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix} .$$

It can be shown that or any pair a, c of co-prime integers there exist integers b and d $ad - bc = 1$ and so all branes are in the $SL(2,\mathbb{Z})$ orbit of the S9-brane. The same argument applies to (p, q)- strings and (p, q)- 5- branes.

There are no other conjugacy classes of half-supersymmetric 9-branes.

Let us assume the 9-brane charges are quantized. The S-duality group becomes $SL(2,\mathbb{Z})$.

The $SL(2,\mathbb{Z})$ orbit of the S9-brane of the linear doublet is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix} .$$

It can be shown that or any pair a, c of co-prime integers there exist integers b and d $ad - bc = 1$ and so all branes are in the $SL(2,\mathbb{Z})$ orbit of the S9-brane. The same argument applies to (p, q)- strings and (p, q)- 5- branes. A more complicated argument shows in the non-linear doublet that all 9-branes lie in the $SL(2,\mathbb{Z})$ orbit of the a single D9-brane.

6 – Conclusion

[a] The one associated to Polchinski's open Heterotic strings?

6 – Conclusion

★ The completion of $N = 2B, d = 10$ supergravity has lead to the discovery of a number of 10-form potentials and their S-duality relations.

[a]The one associated to Polchinski's open Heterotic strings?

6 – Conclusion

- ★ The completion of $N = 2B, d = 10$ supergravity has lead to the discovery of a number of 10-form potentials and their S-duality relations.
- ★ We have used supersymmetry of the effective actions to determine the tensions of the associated 9-branes and we have shown that there are only two supersymmetric doublets:

[a]The one associated to Polchinski's open Heterotic strings?

6 – Conclusion

- The completion of $N = 2B, d = 10$ supergravity has lead to the discovery of a number of 10-form potentials and their S-duality relations.
- We have used supersymmetry of the effective actions to determine the tensions of the associated 9-branes and we have shown that there are only two supersymmetric doublets:
  - a linear doublet with tensions g^{-2}[a] $, g^{-3}$.

[a] The one associated to Polchinski's open Heterotic strings?

6 – Conclusion

- The completion of $N = 2B, d = 10$ supergravity has lead to the discovery of a number of 10-form potentials and their S-duality relations.
- We have used supersymmetry of the effective actions to determine the tensions of the associated 9-branes and we have shown that there are only two supersymmetric doublets:
  - a linear doublet with tensions g^{-2}[a] ,g^{-3}.
  - non-linear doublet with tensions $\sim g^{-1}, g^{-4}$ which includes the D9-brane.

[a] The one associated to Polchinski's open Heterotic strings?

6 – Conclusion

- The completion of $N = 2B, d = 10$ supergravity has lead to the discovery of a number of 10-form potentials and their S-duality relations.
- We have used supersymmetry of the effective actions to determine the tensions of the associated 9-branes and we have shown that there are only two supersymmetric doublets:
  - a linear doublet with tensions g^{-2}[a] ,g^{-3}.
  - non-linear doublet with tensions $\sim g^{-1}, g^{-4}$ which includes the D9-brane.
- T-duality requires the existence of $N = 2A, d = 10$ 9-branes. Work under way...

[a] The one associated to Polchinski's open Heterotic strings?

This is

THE END