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Causality in inflationary universes

Memoria de Tésis de Máster realizada por

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A mi padre,
que ya no está.
Y a mi madre,
que siempre está ahí.

Contents

Summary

A period of exponential expansion of the universe in its early stages of evolution, an inflationary epoch, is a basic ingredient of all current cosmological theories.

This inflationary expansion can solve, among others, the horizon problem, related to the homogeneity (and isotropy) of the Cosmic Microwave Background (CMB) radiation, that is a causal problem.

Since solving a causal problem is one of the main motivations of the inflationary scenario, we have explored in this work the idea of using causality considerations to check for the internal consistency of inflationary theories and to constrain them. In terms of internal consistency for example one can investigate whether the inflationary conditions at the onset of inflation need to happen in regions smaller than the particle horizon at that time (what is called local inflation in Vachaspati & Trodden (1999) [21]).

We have searched the literature for authors that work in this direction, and we have found that yes, causality can put constraints on how the exponential expansion has to proceed, for example on when the decay of the inflaton field has to start (Ellis & Stoeger 1988) [12] or on when the inflationary epoch itself has to begin (Ellis *et al.* 2002)[18].

We have also found that the study of the embedding of the inflationary region within its background non-inflationary spacetime can complement the causal consistency question, in the sense that good embeddings, respecting the weak energy condition, can be found for fully causally produced initial inflationary regions, that is, in the conditions of local inflation (Berera & Gordon 2001) [22].

On the other hand, given the future eternal nature of the inflationary process, Borde, Guth and Vilenkin (2003)[23] asked themselves whether it could also be eternal in past directions. We have finally reproduced their calculations, that show that inflationary spacetimes with positive average expansion rates are singular, with incomplete causal geodesics, and therefore not past eternal.

Introduction

During the last decades cosmology has become an also observational branch of physics: thanks to a handful of fully cosmologically-oriented experiments we can now not only theorize about our universe, but also confirm or discard observationally our theories.

The current *standard cosmological model* [1], [2] tells us that our universe started around 14 Gyr ago from an initial singularity[1] and after a short inflationary period continued expanding (and cooling down) at a slower rate until around 5 Myr ago [3] when its expansion started to accelerate again. The matter-energy content is about 4% of standard baryonic matter, $\Omega_b \sim 0.04$, 21% of cold dark matter, $\Omega_{cdm} \sim 0.21$, and the rest 74% of dark energy, $\Omega_\Lambda \sim 0.74$. The spatial geometry is very close to flat, $\Omega_k \sim 0$, and the current value for the Hubble constant is $H_0 \sim 74 km\ s^{-1} Mpc^{-1}$.

These values for the parameters comprise the so-called *concordance model*, that has been constructed to encompass the following (non-complete) set of cosmological data:

- The Cosmic Microwave Background (CMB) radiation, that tells us that the universe was homogeneous and isotropic to a precision of 10^{-5} in temperature in current distance scales of 100 Mpc and larger [4].

- Deep galaxy surveys, that probe the large scale structure of the baryonic matter content of the universe [5].

- Distances to type Ia Supernovae, that provide independent distance scale estimations and probe the current accelerated expansion [3].

- Dynamics of galaxies (like rotational curves) and clusters of galaxies, that help quantify the dark matter content of the universe.

[1] We should rather say from an initial state with extremely high temperature, density and curvature that cannot be described properly with neither General Relativity (GR) nor Quantum Field Theory (QFT); a state therefore beyond what current physical theories can tackle.

- Gravitational lensing experiments, that give independent measurements of the (dark) matter content.

The early inflationary period, *inflation*, has become crucial to explain some of these experimental facts, and also to explain, for example, why we do not observe the massive magnetic monopoles predicted by Grand Unification Theories [4].

Among the observational features inflation can explain, the most important are the homogeneity (and isotropy) of the CMB radiation and the spatial flatness of the universe. The *Hot Big Bang theory* (HBB), without inflation, that was the standard cosmological model in the 80s, could by no means explain any of the two.

There are, of course, alternatives to standard inflation, like, for example, the cyclic cosmologies of Steinhardt & Turok (2002) [6], or the island cosmology of Dutta & Vachaspati (2009) [7]. Even regular cosmologies (without initial singularity) are postulated by numerous authors, like the bouncing universes of Novello & Bergliaffa (2008) [8], the ekpyrotic universes of Lehners (2008) [9] (where the singularity is in fact a collision of branes that is preceeded by an ekpyrotic slowly contracting phase), the emergent universes of Ellis & Maartens (2004) [10] that undergo no quantum gravity era, etc.

All these alternatives, however, include in their formulations an inflationary epoch as an unavoidable ingredient, so the study of inflation is a basic question.

The problem of explaining the large-scale homogeneity and isotropy of the CMB radiation within the framework of the HBB cosmology, is a causal problem. The CMB photons were produced during *recombination*, an epoch of the early universe in which the plasma was cool enough as to allow free electrons to recombine with the nuclei already present (*recombination* is posterior to the Big Bang nucleosynthesis phase). Sometime afterwards, the mean free path of photons became larger than the size of the universe at that moment, so they decoupled from the rest of the plasma, during *decoupling* [11].

The *Last Scattering Surface (LSS)* is the region where the CMB photons come from, and whose physical properties they map. When we look at those photons, whose interaction all the way down to us with the rest of the universe has been quite scarce, we see that their (microwave) spectrum is homogeneous and isotropic to a level of precision of 10^{-5} in $\Delta T/T$, telling us that the LSS had the same degree of homogeneity.

When you compute backwards in time the sizes of the current homogeneous regions under the standard HBB expansion history, given by the HBB scale factor, $a(t)$, you find that these regions are much bigger than the size of the *particle horizon* of the universe at that time. So, if the last scattering surface is not causally connected, how can it be homogeneous? This question is often refered to as the *horizon problem*.

Therefore, since solving a causal problem is one of the main motivations of inflation, it is important to make sure that acausal phenomena or initial conditions are not required in the inflationary scenario. In other words: causality can be used to check for the internal consistency of inflationary theories.

Some authors work in this direction. For example Ellis & Stoeger (1988) [12] analysed in detail the horizon problem and explained how the inflationary exponential expansion could solve it. But they also pointed out that, in order to respect causality, the decay of the inflaton field must occur not only at the end of the inflationary phase (as was suggested in 1981 in *old inflation* [13]), but rather during the whole inflationary epoch. This way, *the problem of graceful exit from inflation* could be solved.

The problem is the following: if the decay starts close to the end of inflation, due to its stochastic nature, some regions within the inflationary patch will decay (thermalize) and some others will not. The short period of time from there to decoupling will prevent the thermalized regions to get causally connected to the rest, so we find again the horizon problem but this time at the end of inflation.

Subsequent inflationary theories, like *new inflation* [14], [15] or *chaotic inflation* [16], [17] required the decay to start soon after the onset of inflation, so causal connection could have time to proceed and homogeneize the whole last scattering surface.

In a more recent work, Ellis *et al.* (2002) [18] have pointed out that HBB postinflationary cosmologies with positive curvature, only solve the horizon problem if inflation starts close to the Planck era, with extremely high values of Ω_Λ. Otherwise, the existence of *event horizons*, prevents again the homogeneization of the whole LSS.

Although spatially flat cosmologies are most commonly accepted as the ones describing best our universe, the fact is that current cosmological data are also compatible with non-flat cosmologies, with non-critical total energy densities, $\Omega_{tot} \neq 1$ [19].

Event horizons are crucial ingredients of inflationary theories. As we will define in section 1.8 the event horizon of any event of a spacetime is the surface at the proper distance light would travel from that event to $t \to +\infty$. If this distance is bounded, as in the case of exponential expansion, we have causal horizons that isolate the events within them from those in the outside. Inflationary spacetimes are therefore composed of causally disconnected regions that evolve in an independent way [20].

Vachaspati & Trodden (1999) [21] and Berera & Gordon (2001) [22] are the other authors we have found in the literature that deal with these questions. They both talk about *local* *inflation*, meaning that the inflationary conditions that trigger the exponential expansion must occur in a patch of the background spacetime that does not exceed the size of its causal border at that time. This way, inflation proceeds from an initial patch of the universe that is causally connected.

The question they study is whether the *embedding* of the inflationary patch within its background sorrounding spacetime is well behaved, in the sense that positive energies are measured in the boundary of the two spacetimes.

The first authors [21] analyse only the flat case, $\Omega_{tot} = 1$, and find that initial non-local (or acausal) homogeneity is a necessary condition for the embedding to be well behaved, finding therefore an inconsistency in the inflationary scenario. Berera & Gordon [22] however, who generalize the study to any initial global geometry, any initial Ω_{tot}, find that local inflation can actually occur if $\Omega_{tot} < 1$ at the onset of inflation.

We see that causal arguments can actually constrain inflationary theories, that is the idea we are investigating in this work. Our aim is to introduce ourselves in this area, by studying carefully causality issues in General Relativity (in chapter 1) and by learning the fundamentals of the inflationary scenario and exploring the literature on the specific topic of causality in inflation (in chapter 2).

As a mainly introductory work, we will not be presenting results, but rather the tools, concepts and questions that we would like to address as the natural continuation of the work. As part of the introduction we have reproduced (in section 2.3) the work by Borde, Guth and Vilenkin (2003) [23] on the non-past eternal nature of inflation.

Chapter 1

Causal structure

We review in this chapter the language and techniques we will use in this work, that are those of causality in general 4D spacetimes. We will be dealing with pseudo-Riemannian[1] manifolds, $\mathcal{M}$, endowed with metrics g_{ab} of Lorentzian signature[2] that do not necessarily obey Einstein's field equations. $(\mathcal{M}, g_{ab})$ is what we call a **spacetime**.

As well as the causal aspects, we are also interested in identifying and studying the geometrical entities and tools relevant for cosmology, in particular for example we are interested in non-connected spacetimes, which, as we will see in chapter 2 (see section 2.2), are produced in the inflationary phase of the universe.

The simplest definition of a **connected spacetime** is that it cannot be written as the union of two disjoint open subsets. For our purpose however, the concept of **arcwise connected** is more suitable: a spacetime where any two events can be connected by a continuous arc (map or application).

The main references for this chapter are Nakahara [24], Wald [25], Hawking and Ellis [26], Penrose [27], Geroch [28] and Poisson [29].

Vector field; integral curves; flow

Let's recall first that $F(\mathcal{M})$ is the set of continuous real functions on $\mathcal{M}$:

$$F(\mathcal{M}) = \{f : \mathcal{M} \to \mathbb{R} \; / \; f^{-1}[(a,b)] \; \texttt{is an open set of} \; \mathcal{M}\}\,. \tag{1.1}$$

A **vector field** $V = V^\mu \partial_\mu$ is the set of nonvanishing vectors of continuous components $V^\mu(x)$ that live in $\mathcal{M}$. More technically, $V = V^\mu \partial_\mu$ is a vector field if $V(f)$ is also continuous for any continuous function f of $F(\mathcal{M})$.

[1] with symmetric metrics for which if $g_{ab}X^aY^b = 0 \; \forall \; Y \Rightarrow X = 0$

[2] We use $(-,+,+,+)$ in this work

The **integral curves** $x^\mu(\lambda)$ of a vector field V are the curves whose tangent vectors are those of V at each point p of the curve: $V^\mu|_p = \frac{dx^\mu(\lambda)}{d\lambda}|_p$. Each integral curve is parametrized by a parameter λ, that can be chosen to be the affine parameter. They never intersect with each other.

The vector field is said to generate the integral curves, and viceversa.

And, finally, the **flow** generated by V is the application or map, Φ, that produces all the integral curves, starting from the initial points $\{P_0 = x^\mu(0)\}$ of each curve and letting the parameter λ run:

$$\begin{aligned} &\Phi : \mathbb{R} \times \mathcal{M} \rightarrow \mathcal{M}\,, \\ &(\lambda, P_0) \mapsto \Phi(\lambda, P_0) \equiv \Phi_\lambda(P_0) = x^\mu(\lambda)\,. \end{aligned} \tag{1.2}$$

The relationship between the flow and the vector field is the following: $V^\mu|_p = \frac{d}{d\lambda}\{\Phi_\lambda[x^\mu(P_0)]\}$, since $V^\mu|_p = \frac{dx^\mu(\lambda)}{d\lambda}|_p$, and $x^\mu(\lambda) = \Phi_\lambda(P_0) = \Phi_\lambda[x^\mu(P_0)]$.

For a given value of λ, $\Phi_\lambda(P_0)\ \forall\ P_0$ is a diffeomorphism of $\mathcal{M}$ into $\mathcal{M}$ that changes from curve to curve within the family of integral curves. Being a **diffeomorphism** means that $\Phi_\lambda(P_0)$ is differentiable and with inverse also differentiable.

Parallel transport; connection; covariant derivative

In a curved spacetime it is not possible in general to cover the whole manifold with a unique chart/coordinate system. Rather, there is an atlas to cover it and local tangent spaces on every point of the manifold. The charts are homeomorphic with open subsets of Minkowski spacetime, that is, they can be continously deformed into open flat subsets.

The structure therefore is quite more complex than in the flat Euclidean, or Minkowskian, case, and the usual directional derivative, ∂_μ, cannot be used to calculate how vector or tensor fields vary on the manifold along the directions given by the (local) coordinate systems.

In order to do so, we have to **parallel transport** the vector or tensor field along these directions and see how it varies. For clarity we continue the explanation for the simpler case of a vector field, $V^\nu(x)$, whose infinitesimal change following the rule of the parallel transport is the following:

$$V^\nu(x) \rightarrow V^\nu(x + \Delta x^\mu) = V^\nu(x) + \Delta x^\mu \Gamma^\nu_{\mu\delta} V^\delta\,. \tag{1.3}$$

The operator $\Gamma^\nu_{\mu\delta}$ is a non-tensor called the **connection** that gives us how each vector of the coordinate basis $e_\mu = \partial_\mu$ varies along the other coordinate directions, $\{e_\nu = \partial_\nu, \nu \neq \mu\}$:

$$\nabla_\nu e^\delta_\mu \equiv \nabla_{e_\nu} e^\delta_\mu = \partial_\nu e^\delta_\mu + \Gamma^\delta_{\nu\lambda} e^\lambda_\mu = \partial_\nu \delta^\delta_\mu + \Gamma^\delta_{\nu\lambda} e^\lambda_\mu = \Gamma^\delta_{\nu\mu}\,, \tag{1.4}$$

where μ in e_μ just labels the vector of the coordinate basis we are working with, and not its coordinate.

$\Gamma^\nu_{\mu\delta}$ is called a connection (between tangent spaces) because the vectors equivalent to those of $T_p(\mathcal{M})$ in another $T_q(\mathcal{M})$ are constructed parallel transporting them with $\Gamma^\nu_{\mu\delta}$ from $T_p(\mathcal{M})$ to $T_q(\mathcal{M})$.

If we moved $V^\nu(x)$ to $x + \Delta x^\mu$ without parallel transporting it, but simply expressing its components in that local coordinate system, the one at $x + \Delta x^\mu$, we would get the vector field $\widetilde{V}^\nu(x + \Delta x^\mu)$. Now, the **covariant derivative** is the limit when $\Delta x^\mu \to 0$ of the difference between this vector field and that obtained making the parallel transport:

$$\nabla_\mu V^\nu = \lim_{\Delta x^\mu \to 0} \frac{\widetilde{V}^\nu(x + \Delta x^\mu) - V^\nu(x + \Delta x^\mu)}{\Delta x^\mu} = \partial_\mu V^\nu + \Gamma^\nu_{\mu\delta} V^\delta \,, \tag{1.5}$$

that is the usual/Euclidean directional derivative $\partial_\mu V^\nu$ plus the term $\Gamma^\nu_{\mu\delta} V^\delta$ that takes into account the non-trivial parallel transport. The covariant derivative, $\nabla_\mu V^\nu$, tells us how V^ν varies along any coordinate direction ∂_μ.

The fundamental theorem of the pseudo-Riemannian geometry states that there is a unique symmetric connection Γ compatible with the metric, and it is called the Levi-Cività connection. Compatibility means that for any pair of vector fields V and W in $\mathcal{M}$, their scalar product $g(V, W) = g_{ab} V^a W^b$ remains constant when parallel-transporting them along any curve in $\mathcal{M}$ (the metric is covariantly constant: $\nabla_\mu g_{\nu\sigma} = 0$).

1.1 Causal curves

A differentiable curve $c(\lambda)$ is a **past causal curve** when its tangent vector field T is timelike or null past-directed (see section 1.2):

$$T^\mu = \frac{dc(\lambda)}{d\lambda}|_p = \frac{dx^\mu(\lambda)}{d\lambda}|_p \neq 0 \texttt{ is timelike or null past-directed.} \tag{1.6}$$

A differentiable curve $c(\lambda)$ is a **future causal curve** when its tangent vector field T is timelike or null future-directed:

$$T^\mu = \frac{dc(\lambda)}{d\lambda}|_p = \frac{dx^\mu(\lambda)}{d\lambda}|_p \neq 0 \texttt{ is timelike or null future-directed.} \tag{1.7}$$

Inextendible causal curves

Let $c(\lambda)$ be a causal past or future curve. A point $p_{end} \in \mathcal{M}$ is a **past or future endpoint** of $c(\lambda)$ if the curve never gets out of any neighborhood of p_{end} once it gets into it. Formally, p_{end}

is a past or future endpoint of $c(\lambda)$ if $\forall\ O_p$ neighborhood of $p_{end}\ \exists\ \lambda_0\ /\ c(\lambda) \in O_p\ \forall\ \lambda > \lambda_0$. The endpoint does not neccessarily belong to the curve.

A curve that does not have a past or future endpoint is called an **inextendible past or future curve**. Inextendible here must be understood as endless, without endpoint, inextendible because the curve is as extended as it can be.

1.2 Geodesics

Among the causal curves, we are most interested in the worldlines of free-falling particles: the timelike and null geodesics. Let $(\mathcal{M}, g_{ab})$ be our spacetime, and let $p, q \in \mathcal{M}$ be two points of $\mathcal{M}$ with their corresponding tangent spaces $T_p(\mathcal{M})$ and $T_q(\mathcal{M})$.

Let Γ be the Levi-Cività connection parallel-transporting vector fields from $T_p(\mathcal{M})$ to $T_q(\mathcal{M})$ and let $\gamma(\lambda)$ be a curve in $\mathcal{M}$, $x^\mu(\lambda)$ in some coordinates. The tangent vector field to $\gamma(\lambda)$ is $T^\mu = \frac{d}{d\lambda}x^\mu(\lambda)$. The curve $\gamma(\lambda)$ is called a **geodesic** when its tangent vector field T is parallel-transported along $\gamma(\lambda)$, i.e., $\nabla_T T = 0$, that in components are the so-called geodesic equations:

$$\begin{aligned} \frac{dT^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} T^\alpha T^\beta &= 0\,, \\ \frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda} &= 0\,. \end{aligned} \tag{1.8}$$

where λ is the affine parameter of the curve for which actually $\nabla_T T = 0$.

These differential equations have a unique solution for a given set of initial conditions so $\forall\ p \in \mathcal{M}$ and $\forall\ t \in T_p(\mathcal{M})$ there exists a unique solution to them, or to put it another way, $\forall\ p \in \mathcal{M}$ there exists a bundle of geodesics emanating from it generated by the vectors of the tangent space in p.

The geodesic that joins the points $p, q \in \mathcal{M}$ is the curve that extremizes their arclength, l:

$$l = \int_c (\pm g_{ab} T^a T^b)^{1/2} d\lambda\,. \tag{1.9}$$

Incomplete geodesics

An inextendible causal geodesic in at least one direction (future or past) is said to be an **incomplete geodesic** when its affine parameter in that direction is bounded. Spacetimes with one incomplete geodesic are called **singular spacetimes**. The incomplete geodesic can be either spacelike, null or timelike, so a spacetime with only spacelike incomplete geodesics can not be detected as such.

Recalling that inextendible in this context means endless, incomplete causal geodesics are timelike or null geodesics that extend all along the manifold without getting into endpoints, but still with finite values for their affine parameters.

Null cone; light cone

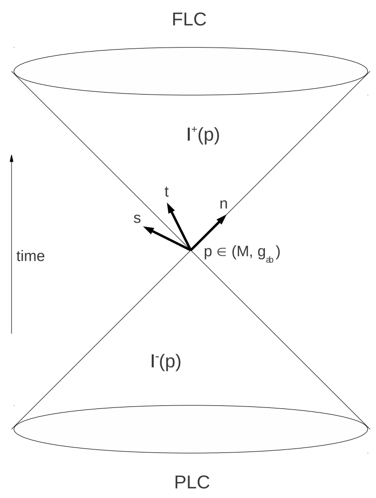

Figure 1.1: Null cone of $p \in (\mathcal{M}, g_{ab})$.

The **null cone** of p, also-called light cone by some authors, is the usual light cone of special relativity (SR), that is isomorphic to the tangent space $T_p(\mathcal{M})$ in p. The null cone is therefore populated by vectors of the tangent space, not by events q of the manifold.

Spacelike (s), timelike (t) and null (n) vectors are defined as in figure 1.1 and for a metric of signature (-,+,+,+) their length is positive, negative and null respectively:

$$s^2 = s^\mu s_\mu > 0\,, \tag{1.10}$$

$$t^2 = t^\mu t_\mu < 0\,, \tag{1.11}$$

$$n^2 = n^\mu n_\mu = 0\,. \tag{1.12}$$

FLC and PLC stand for future light cone and past light cone, and the regions inside them are the chronological future, $I^+(p)$, and past, $I^-(p)$ of p, as defined in section 1.4.

The **light cone** of the event (or point) p of the manifold $\mathcal{M}$ is the subset of $\mathcal{M}$ generated by all the null geodesics that emanate from p. Unlike the null cone, the light cone is indeed populated by events q of the manifold.

1.3 Congruences of geodesics

Let $O \subset \mathcal{M}$ be an open subset of the spacetime $(\mathcal{M}, g_{ab})$. A **congruence of geodesics** $\gamma_s(\lambda)$ in O is a one-parameter family of geodesics with affine parameter λ with nonvanishing tangent vector field T. It can also be defined as a family of integral curves of a vector field T such that through each $p \in O$ only one of the curves of the congruence passes. The map $(\lambda, s) \mapsto \gamma_s(\lambda)$ is smooth, one-to-one and of smooth inverse.

$\gamma_s(\lambda)$ creates a 2D surface in the manifold with coordinates $\{\partial_\lambda, \partial_s\}$:

$$
\begin{aligned}
T &= \frac{d\gamma_s(\lambda)}{d\lambda} \quad \texttt{is the tangent vector field} \\
X &= \frac{d\gamma_s(\lambda)}{ds} \quad \texttt{is the deviation vector field}
\end{aligned}
\tag{1.13}
$$

The **congruence deviation field** X is the solution to the **geodesic deviation equation**:

$$
a^\mu = -R^\mu{}_{\nu\sigma\lambda} X^\sigma T^\nu T^\lambda, \tag{1.14}
$$

with $a^\mu = \nabla_T(\nabla_T X^\mu)$ and $R^\mu{}_{\nu\sigma\lambda}$ the Riemann curvature tensor. The quantity $v^\mu = \nabla_T X^\mu$ measures the variation of the relative distance among geodesics along their trayectories, and therefore $a^\mu = \nabla_T(\nabla_T X^\mu)$ is the relative acceleration among them. The geodesic deviation equation is obtained from the definition of a^μ by simply realizing that $\nabla_T(\nabla_T X^\mu)$ is related to the action of the Riemann tensor on the vector fields T^μ and X^μ. The solutions to this equation are called **Jacobi fields**.

1.3.1 Expansion scalar, Raychaudhuri equation

The **expansion scalar**, θ, measures the mean expansion of infinitesimally close geodesics, that is the fractional change of the volume subtended by the geodesics of the congruence per unit affine parameter along them.

To define it we have to start by defining the tensorial field B_{ab}:

$$
B_{ab} \equiv \nabla_b T_a \tag{1.15}
$$

with T^a the timelike unitary ($T_a T^a = -1$) tangent vector field of the congruence. B_{ab} is purely spatial, $B_{ab}T^a = B_{ab}T^b = 0$, and has the following physical interpretation: $\nabla_T X^b = B^b_a X^a$ and therefore B_a^b measures how nonparallel is the transport of X^b along the congruence (X^b is the deviation vector field of the congruence). An observer following the geodesics would find their sorrounding geodesics diverging, converging, twisting or rotating due to the action of the map B_{ab}. If $B_{ab} = 0$ then the deviation vectors are parallel-transported along the congruence and the observer would see their sorroundings always staying the same.

And now we need to define the extended metric h_{ab}:

$$h_{ab} = g_{ab} + T_a T_b \,, \tag{1.16}$$

so that $h_b^a = g^{ac} h_{cb}$ is the operator that projects the tangent vector field T^a into its perpendicular subspace.

Now we can define the expansion θ:

$$\theta \equiv B^{ab} h_{ab} \,. \tag{1.17}$$

From its definition and using the geodesic deviation equation (1.14) we can find the dynamical equation of θ, the so-called **Raychaudhuri equation**:

$$T^c \nabla_c \theta = \frac{d\theta}{d\lambda} = -\frac{1}{3}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - R_{cd} T^c T^d \tag{1.18}$$

where R_{cd} is the Ricci tensor, σ_{ab} is the shear, that measures how an initial sphere would get distorted into an ellipsoid, and ω_{ab} is the antisymmetric part of B_{ab} that measures the rotation of the geodesics of the congruence. This equation is crucial for the singularity theorems.

1.4 Causal past and future

The **causal past**, $J^-(p)$, of an event $p \in \mathcal{M}$ is the set of points of the manifold that can be reached from p with past-directed causal curves, that is, with past-directed null or timelike curves:

$$J^-(p) \equiv \{ q \in \mathcal{M} / \; \exists \; c(\lambda) \; \texttt{timelike or null past-directed} \; / p, q \in c(\lambda) \; \} \,. \tag{1.19}$$

The **causal future**, $J^+(p)$, of an event $p \in \mathcal{M}$ is the set of points of the manifold that can be reached from p with future-directed causal curves, that is, with future-directed null or timelike curves:

$$J^+(p) \equiv \{ q \in \mathcal{M} / \; \exists \; c(\lambda) \; \texttt{timelike or null future-directed} \; / p, q \in c(\lambda) \; \} \,. \tag{1.20}$$

When only timelike curves are considered to join p, we are dealing with the **chronological past** and **future** of p:

$$I^-(p) \equiv \{ q \in \mathcal{M} / \; \exists \; c(\lambda) \; \texttt{timelike past-directed} \; / p, q \in c(\lambda) \; \} \,, \tag{1.21}$$

$$I^+(p) \equiv \{q \in \mathcal{M}/ \ \exists \ c(\lambda) \ \texttt{timelike future-directed} \ /p, q \in c(\lambda) \ \} \,. \tag{1.22}$$

Since it is always possible to continously deform a timelike curve into another one that ends within a neighborhood of q, $I^-(p)$ and $I^+(p)$ are always open sets.

For a subset $S \subset \mathcal{M}$ its causal past and future are defined:

$$J^-(S) = \cup_{p \in S} \ J^-(p) \,, \tag{1.23}$$

$$J^+(S) = \cup_{p \in S} \ J^+(p) \,, \tag{1.24}$$

i.e., simply the union of the individual pasts and futures of all the points in S. Similarly the chronological past and future of S are constructed with the union of the individual ones:

$$I^-(S) = \cup_{p \in S} I^-(p) \,, \tag{1.25}$$

$$I^+(S) = \cup_{p \in S} I^+(p) \,, \tag{1.26}$$

which are also always open sets.

Convex normal neighborhood

A neighborhood of an event p, U_p, is called a **convex normal neighborhood** when every two events in U_p are joined by a unique geodesic that does not leave U_p.

Formally, U_p is a convex normal neighborhood when $\forall \ q, r \in U_p \ \exists! \ \gamma(\lambda)$ geodesic that joins q and r staying entirely within U_p.

The chronological future of U_p, $I^+(U_p)$, is the set of points $q \in \mathcal{M}$ which are linked to p by timelike future geodesics contained in U_p. Therefore $I^+(U_p) \subset U_p$.

For a convex normal neighborhood, $I^+(U_p)$ and its boundary $\partial I^+(U_p)$ are generated by timelike and null geodesics respectively, as in Minkowski spacetime. This makes this object crucial to give any spacetime $(\mathcal{M}, g_{ab})$, and locally, the causal structure of Minkowski spacetime.

Achronal set and its edge

A subset of $\mathcal{M}$, $A \subset \mathcal{M}$ is called **achronal** if none of the points in A belongs to the chronological future of any of the rest of the events in A. Formally, $A \subset \mathcal{M}$ is an achronal set if $\nexists \ p, q \in A \ / \ q \in I^+(p)$, i.e., if $I^+(A) \cap A = 0$.

The **edge** of A, $edge(A)$, is the set of points of A that have for every neighboorhod around them one event in their chronological future and one event in their chronological past that can

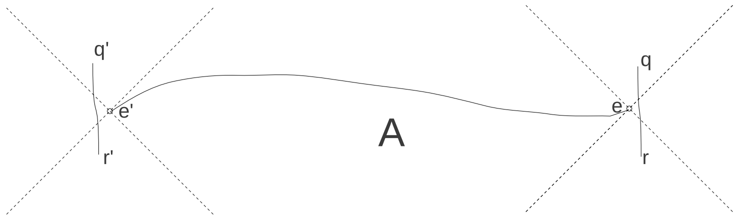

Figure 1.2: A represents an achronal subset of a conformally flat spacetime, $\mathcal{M}$, where null worldlines are straight lines at $\pm 45°$. The points e and e' are the edge of A.

be joined by a timelike curve that does not intersect A; that lies entirely outside A. That is, the set of points $p \in A / \ \forall \ O_p$ open neighborhood of p, $\exists \ q \in I^+(p)$ and $\exists \ r \in I^-(p)$ which are joined by a timelike curve that does not intersect A.

See in fig. 1.2 how $\forall p \notin edge(A)$ it is not possible to join q and r without intersecting A.

1.5 Domains of dependence

While $I^{\pm}(S)$ and $J^{\pm}(S)$ are the collections of points of the manifold that can be causally related to the subset $S \subset \mathcal{M}$, it is also interesting to define the collections of points of the manifold which are causally *completely determined* by a given subset S of the manifold. These are the domains of dependence of S, $D^{\pm}(S)$.

The past domain of dependence of the subset $S \subset \mathcal{M}$, $D^-(S)$ is the collection of points $p \in \mathcal{M}$ whose future inextendible causal curves, all of them, intersect S:

$$D^-(S) \equiv \{p \in \mathcal{M} \ / \ \texttt{every future inextendible causal curve through p intersects} \ S\} \, . \tag{1.27}$$

The future domain of dependence of the subset $S \subset \mathcal{M}$, $D^+(S)$ is the collection of points $p \in \mathcal{M}$ whose past inextendible causal curves, all of them, intersect S:

$$D^+(S) \equiv \{p \in \mathcal{M} \ / \ \texttt{every past inextendible causal curve through p intersects} \ S\} \, . \tag{1.28}$$

Therefore, the physical conditions in S completely determine the physical conditions in $D^+(S)$ and the physical conditions in $D^-(S)$ determine those of S.

The (total) domain of dependence is the union of both the past and future ones: $D(S) = D^+(S) \cup D^-(S)$.

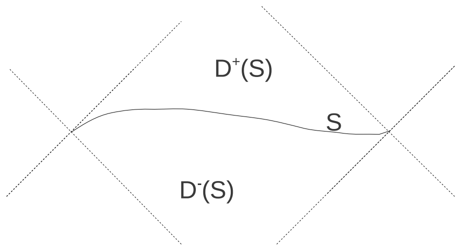

Figure 1.3: These are the future, $D^+(S)$, and past, $D^-(S)$, domains of dependence of the subset $S \in \mathcal{M}$. As in figure 1.2 consider $\mathcal{M}$ a conformally flat spacetime.

Cauchy surface

A closed achronal subset $A \subset \mathcal{M}$ for which $D(A) = \mathcal{M}$, i.e., that determines completely the physical conditions of the whole manifold, is called a **Cauchy surface**.

It is a *surface* because a theorem proves that closed achronal subsets without edge are (D-1) dimensional submanifolds of $\mathcal{M}$ and C^0 (continuous).

A spacetime $(\mathcal{M}, g_{ab})$ that has a Cauchy surface is called a **globally hyperbolic spacetime**.

1.6 Orientable manifolds

Let $\mathcal{M}$ be a D-dimensional connected manifold covered with an atlas $\{u_i\}$. $\mathcal{M}$ is said to be **orientable** if for any pair of charts u_i, u_j with a non-empty intersection, the Jacobian of the transformation between their coordinates, $\{x^\mu\}$ and $\{y^\nu\}$ respectively, has a positive determinant, $det(\partial x^\mu / \partial y^\nu) > 0$.

To illustrate, the Möbius strip is an example of a non-orientable manifold, since this determinant is negative for the intersection between the two charts you need (at least) to make its atlas.

A manifold in which a continous designation of past and future can be done in the local light cones on every point $p \in \mathcal{M}$ is called a **time-orientable manifold**.

1.7 Causality in manifolds

Spacetimes with nontrivial closed causal curves ($c(\lambda) = p \ \forall \ \lambda$ is the trivial one) are usually considered to be non-physical because they would allow observers to travel back in time. Also those whose causal curves are not closed but are allowed to bend arbitrarily close to themselves are not considered physical either, since a small perturbation of the metric could close the curves. The criteria for good causal behaviour are the following:

A spacetime $(\mathcal{M}, g_{ab})$ is **strongly causal** if $\forall \ p \in \mathcal{M}$ and $\forall \ O_p$ neighborhood of $p \ \exists \ O \subset O_p \ / \ \nexists$ causal curves passing through O more than once. This eliminates the possibility of the curve becoming arbitrarily close to itself.

Let now be the metric $\widetilde{g_{ab}} = g_{ab} - t_a t_b$ defined in $(\mathcal{M}, g_{ab})$ with $t^a \neq 0$ a timelike vector. This metric is also of Lorentz signature and can be thought to broaden the local light cones, since a timelike vector of g_{ab} is also a timelike vector of $\widetilde{g_{ab}}$ and a null vector of g_{ab} is timelike for $\widetilde{g_{ab}}$. We say that $(\mathcal{M}, g_{ab})$ is **causally stable** when a timelike continous vector field t^a exists in $\mathcal{M}$ such that $(\mathcal{M}, \widetilde{g_{ab}})$ contains no timelike closed curves. This is the criterium commonly used to consider a spacetime causally well behaved.

1.8 Miscellanea

In order to homogenize the language and ease discussions and understanding along the text, we introduce here several standard definitions of horizons and related concepts, that are often given in the literature for the particular case of a flat FRWL cosmology, with metric $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$. Here **cosmology** stands for time-orientable spacetime.

One of our objectives is to generalize these definitions for arbitrary cosmologies, with any general $g_{\mu\nu}$, and this way relate them to the objects we have previously formally introduced.

Particle horizon

The **particle horizon** is the surface photons may have reached travelling from the origin of times in a FRWL universe, $t = 0$, to any posterior time t. The comoving radial coordinate of this surface, r_{ph}, is:

$$r_{ph}(t) = c \int_0^t \frac{dt'}{a(t')} \,. \tag{1.29}$$

The universe at $t = 0$ is a particular achronal subset of $\mathcal{M}$, $A_0 \in \mathcal{M}$, that we will call the **initial patch** of the universe. Its causal future is $J^+(A_0)$ and in particular its null future, that we can call $H^+(A_0)$, contains all the particle horizons reached at different t values as the cosmology evolves.

The generalized concept of particle horizon is therefore that at each t an achronal subset of $H^+(A_0)$, $H_t^+(A_0)$, is the particle horizon of the initial patch of the universe, A_0, with the following structure:

$$\begin{aligned} H_t^+(A_0) &= \cup_{p\in A_0} H_t^+(p) \\ H^+(A_0) &= \cup_{\forall t} H_t^+(A_0) \end{aligned} \tag{1.30}$$

See also, recalling the definition of flow at the beggining of the chapter, how we can think of the evolution of a cosmology as a flow that generates the geodesics of A_0 as the cosmological time, t, passes.

Event horizon

The **event horizon** is the surface photons can reach by travelling from a certain t in a FRWL universe up to $t \to +\infty$. The comoving radial coordinate of this surface, r_{eh}, is:

$$r_{eh}(t) = c \int_t^{+\infty} \frac{dt'}{a(t')} \,. \tag{1.31}$$

Recession velocity

The **recession velocity** measures how proper distance $D(t) = \int ds = a(t)r$ among comoving observers in a FRWL universe changes due to the expansion, and not due to local peculiar velocities, $\dot{r}(z) = 0$:

$$v_{rec}(t,z) = \dot{D}(t) = \dot{a}(t)r(z) + a(t)\dot{r(z)} = \dot{a}(t)r(z) = c\,\frac{\dot{a}(t)}{a_0} \int_0^z \frac{dz'}{H(z')} \,, \tag{1.32}$$

since $r(z) = \frac{c}{a_0} \int_0^z \frac{dz'}{H(z')}$. $H(z)$ is the standard Hubble parameter, $H = \frac{\dot{a}}{a}$, and z is the redshift.

Hubble sphere

The **Hubble sphere** is the proper distance, D_{hs}, at which the recession velocity equals the speed of light:

$$v_{rec}(t,z) = \dot{a}(t)r(z) = H(t)D(t) = c \Rightarrow D_{hs} = \frac{c}{H(t)} \,. \tag{1.33}$$

Objects beyond the Hubble sphere receed faster than the speed of light. This fact is not in contradiction with either Special Relativity (SR) or General Relativity (GR) since c as a *local maximum* for the relative velocities among physical objects still holds.

At any given time, objects beyond the Hubble sphere are causally disconnected from those inside it, but since its size changes with the expansion history of the cosmology, they may become causally connected at some other time. In particular, during the HBB evolution,

$$\dot{D}_{hs} = c\left[1 - \frac{a(t)}{\dot{a}(t)^2}\ddot{a}(t)\right] > 0, \tag{1.34}$$

i.e., the Hubble sphere grows, and overcomes previously superluminal regions, making them subluminal. That is why the Hubble sphere is not a horizon, and should not be considered as such. Unfortunately, the term Hubble horizon is vastly used in the literature, leading to confusion [30].

Spacetime diagrams

Spacetime diagrams plot comoving radial distance, $a \cdot r$, in their x axis and conformal time τ in their y axis. Therefore, for a conformally flat spacetime, with metric $ds^2 = a(\tau)^2[-d\tau^2 + dr^2]$, the worldlines of comoving observers are vertical lines of constant r and those of lightrays are straight lines at $\pm 45°$ (see fig. 1.4).

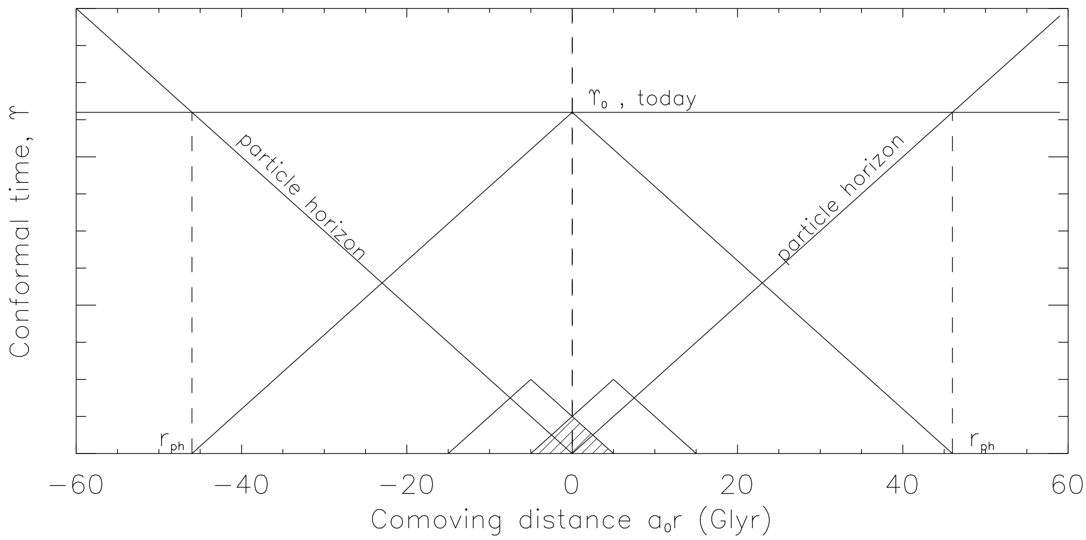

Figure 1.4: Causal border of the universe for us today, $r_{ph}(\tau_0)$.

We have located the comoving observer "us" at $r = 0$ and radial distances are, therefore, measured relative to us. The particle horizon is plotted for the whole history of this observer, whose current conformal time is τ_0 (today). The current size of the particle horizon is $r_{ph}(\tau_0)$, that is the distance photons emitted at $\tau = 0$ would have travelled in their way to us today. See how events inside the particle horizon always share common past regions with us (and among themselves, see shaded area).

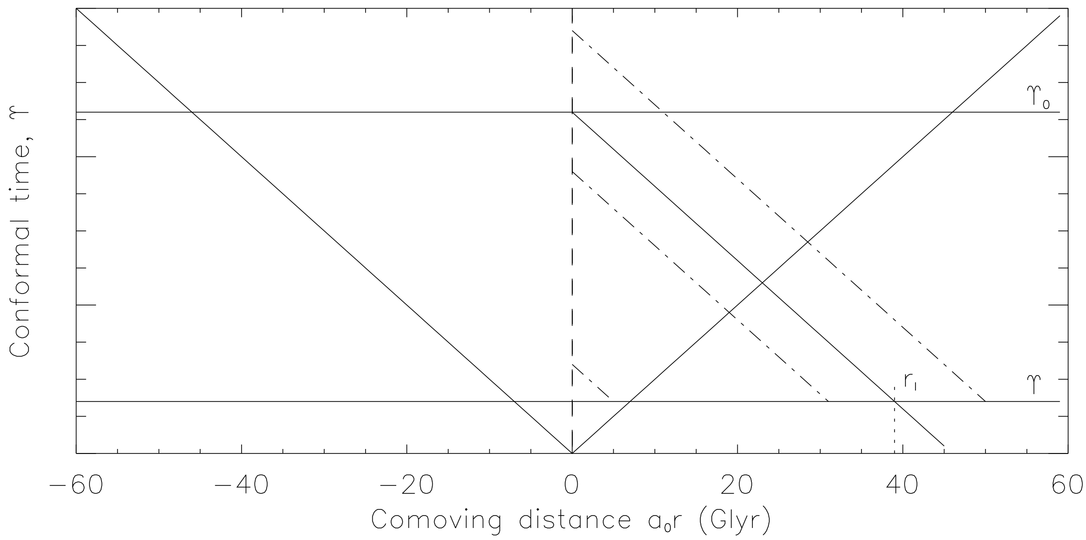

Figure 1.5: Regions of the spacetime accesible to us today are those with $r \leq r_l$.

In figure 1.5 we see one of the characteristic features of these spacetimes. Note that events happening all over the spacetime (for any value of r) at times much earlier than today will not be detected by us unless they happen "close enough": within distances below r_l. Events beyond r_l will affect us sometime in the future (see below about the last scattering surface, r_l).

Last scattering surface as the effective causal border of the universe

As we explained in the introduction, the **last scattering surface** is where the photons of the CMB radiation interacted for the last time with matter, at decoupling, when the age of the universe was around 10^5 yr. Before that, the universe was opaque, that is, the photons' mean free path was quite short, and therefore, radiation never scaped and cannot reach us today [11]. So even though there are no causal limitations to reach $t = 0$, effectively no information is reaching us coming from before decoupling, so the effective causal border of our universe is the last scattering surface. This limit is also called **visual horizon** sometimes [12], see fig. 1.6.

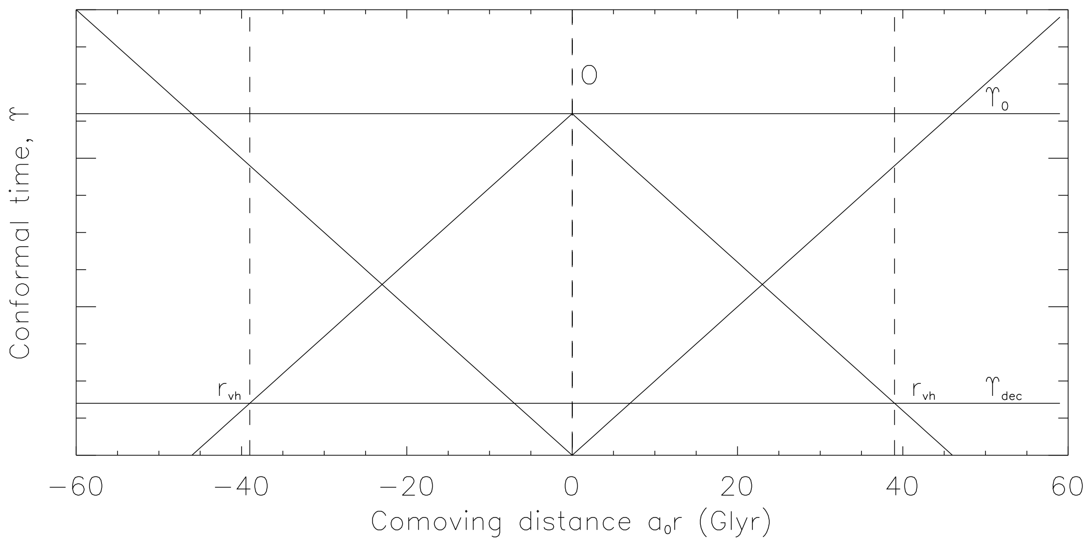

Figure 1.6: Visual horizon, $r = r_{vh}$, today for us, the $r = 0$ observer. For a pure HBB cosmology without inflation.

Chapter 2

Causality issues in inflation

The inflationary paradigm is considered nowadays a good description of the very early evolution of the universe, shortly after the initial singularity. After inflation, the universe continues evolving as the Hot Big Bang (HBB) scenario predicts, into our present Λ-CDM universe: an acceleratingly expanding spatially flat dark energy dominated universe (74%), with cold dark matter (21%) and traces of standard baryonic matter (4%).

Inflationary theories started to be constructed in the early 80s, mainly by A.H. Guth, A.D. Linde, P. Steinhardt and collaborators, to solve some of the major problems in cosmology at that time:

- The horizon problem: the CMB photons decoupled from matter when the age of the universe was around 10^5 yr [20], therefore they map the physical conditions of the universe then. Today, we observe homogeneity in the CMB radiation in regions that at decoupling were not causally connected. How could they share then the same physical conditions?
- The flatness problem: the current value of the total mass-energy density of the universe, ρ_{tot}, relative to the critical density, $\rho_c \equiv 3H^2/8\pi G$, is measured to be $\Omega_{tot} = \rho_{tot}/\rho_c = 1.006 \pm 0.006$ [31]. This implies that the universe is spatially flat, which is an unstable equilibrium point of the Friedmann equations (see equation 2.1 below). This means that any small departure from 1 will quickly grow pulling the universe away from flatness, so, how could it retain the $\Omega_{tot} = 1$ value over its whole evolution?
- The formation of structure: the current observed distribution of baryonic matter (galaxies, clusters and superclusters of galaxies, walls, etc.) points towards a bottom-up formation scenario that needs primordial density pertubartions as seeds for the subsequent aggregation of matter. How are these primordial perturbations formed?
- The absence of magnetic monopoles: all Grand Unified Theories developed in the 70s

predicted the existence of very massive magnetic monopoles, that may have been copiously produced in early epochs of the evolution of the universe, something that is not at all observed.

Although there are individual alternative explanations to all these questions, the main strength of inflation is that it can account by itself for all of them. In order to do so, inflation depicts an initial very high density, ρ, and homogeneous patch of the universe that spontaneously happens to be in a false vacuum state with negative pressure, $p = -\rho$, that triggers the exponential expansion of this patch over the underlying non-inflationary rest of the universe.

This negative pressure, equivalent to a repelling gravity, is responsible for the exponential expansion, that proceeds as many e-folds[1] as necessary (around 60 [21]) to produce the homogeneity and flatness observed.

The simplest way to realize inflation is by considering a massive scalar field, Φ, the inflaton, with a potential of the form $V(\Phi) = \frac{m^2}{2}\Phi^2$[32]. The fluctuations of this field at the end of the inflationary phase are the primordial density perturbations needed for the subsequent formation of structure.

The accelerated exponential expansion brings the total density towards the critical value, making $\Omega_{tot} = 1$. This can be easily seen as follows. The next equation is obtained from the Friedmann equations and gives us the evolution of Ω_{tot} for any expansion history, $a(t)$, so it is valid for both the post-inflationary and inflationary phases. While the post-inflationary (HBB) phase is characterized by a deccelerated expansion, $\ddot{a} < 0$, during inflation $\ddot{a} > 0$, and therefore $|\Omega_{tot}(t) - 1|$ is constantly decreasing. The 60 e-folds bring Ω_{tot} so close to 1 that the subsequent HBB evolution (with $\frac{d}{dt}|\Omega_{tot}(t) - 1| > 0$) can hardly move it away from that value.

$$
\begin{aligned}
|\Omega_{tot}(t) - 1| &= \frac{|k|}{H^2 a^2}\,, \\
\frac{d}{dt}|\Omega_{tot}(t) - 1| &= -2\ddot{a}\frac{|k|}{H^3 a^3}\,.
\end{aligned}
\tag{2.1}
$$

The horizon problem is solved by inflating the initial homogeneous region into a much bigger one in a way that preserves its homogeneity (for more details see section 2.1 below). And finally the monopole problem is solved because inflation dilutes the monopole density to extremely low values [4].

So we have a small patch of the universe that starts expanding exponentially soon after the initial singularity. In the *chaotic inflation* scenario [20] the inflaton is rolling down its potential

[1] An e-fold is the lapse of time in which the universe has multiplied its size by a factor of e.

while expanding in its way to *reheating*, that is the epoch just after inflation, where the field oscillates around its minimum to produce all the particles for the subsequent HBB evolution. We say, then, that the inflationary patch has thermalized and produced a *pocket universe*, not a *bubble* universe to avoid the spherical symmetry the word *bubble* suggests [4].

While still inflating, a region of the inflationary patch can suffer an upward quantum fluctuation of Φ that makes it recover a high value of the energy density that enables inflation to restart in that region. This is how *chaotic inflation* produces many inflationary patches and pocket universes once it starts. And with a fractal structure [32].

In *new inflation* however [4], the inflationary patch continues expanding forever and regions of it continuous and stochastically thermalize, producing the flux of pocket universes that is characteristic of new inflation.

Note the difference between both scenarios: new inflation produces a flux of pocket universes coming from a unique eternal inflationary patch while chaotic inflation produces a pocket universe out of every inflating patch that as well produces another inflationary patch, that in turn will produce a pocket universe and another inflationary patch, etc.

This is why inflation is future eternal and produces not only a pocket universe, but a flux of them, a so-called *multiverse*. While being eternal in the future, it was shown in ref. [23] that it cannot be eternal in the past. So the question of what was there before inflation is still open.

After this brief introduction, in section 2.1 we start analising in detail the *horizon problem* since it is a basic causality issue in inflation. Section 2.2 explains why inflation produces pocket universes populated with causally non-connected regions, and section 2.3 reproduces the theorem leading the authors of ref. [23] to conclude that inflation is not past eternal. Finally, in section 2.4 we see how the study of the embedding of the inflationary spacetime within its background can constrain the cosmological parameters.

2.1 The horizon problem

The so-called **horizon problem** can be explained as follows. The detailed measurements of the CMB radiation starting in the 90s with COBE [33] and improved later on with WMAP [34] and PLANCK [35] have shown that the universe was extremely homogeneous (and isotropic) when this radiation was formed, or more precisely when the CMB photons decoupled from matter, at decoupling ($t_{dec} \simeq 3 \cdot 10^5$ years [11]). This homogeneity amounts to $\Delta T/T = 10^{-5}$.

The present sizes of the homogeneous regions are of the order of 100 Mpc [4], regions that when traced back to decoupling with the usual expansion history of the HBB universe (for example that in [36]) happen to be bigger than the particle horizon at that epoch.

The question arises here: how is it possible that regions that are not causally connected at the epoch of decoupling can share identical physical conditions?

In terms of causality the horizon problem is clearly explained in the following spacetime diagram (see fig. 2.1).

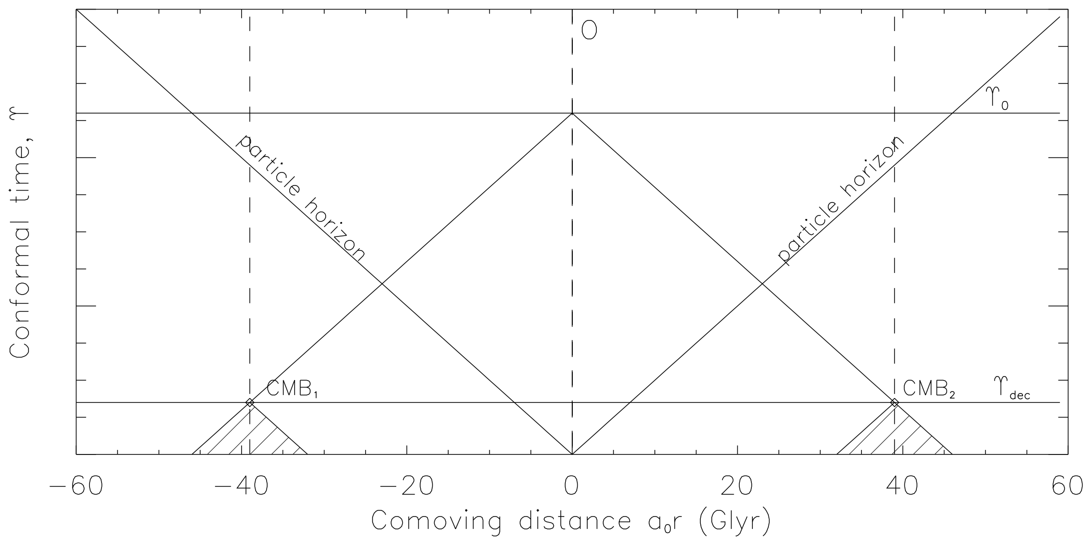

Figure 2.1: Spacetime diagram for a HBB conformally flat universe without inflation. We are the observer O at $r = 0$. $\tau_{\rm dec}$ is the epoch of decoupling and τ_0 is today.

The points CMB_1 and CMB_2 of the last scattering surface that we can see today, are well outside our particle horizon at decoupling and therefore do not share any events in their past light cones (see shaded areas). There can be then no physical reason for them having the same temperature, unless acausal physical mechanisms are invoked.

When an inflationary epoch is included before decoupling the situation is solved as seen in fig. 2.2. Now CMB_1 and CMB_2 are well inside our particle horizon at decoupling, so they have causal connection with us. They also share a common past region (see shaded area) that according to Ellis & Stoeger (1988) [12] can explain their identical physical conditions. However, they both have independently causal contact with regions other than their past common region so in order to justify their claim, the authors of [12] had to determine the volumes of these independent regions in comparison with that of the common past region, finding the latter much bigger than the former.

Another interesting aspect of causality within the inflationary idea, as pointed out also in [12], is that due to its stochastic nature, the decay of the inflaton cannot happen close to the

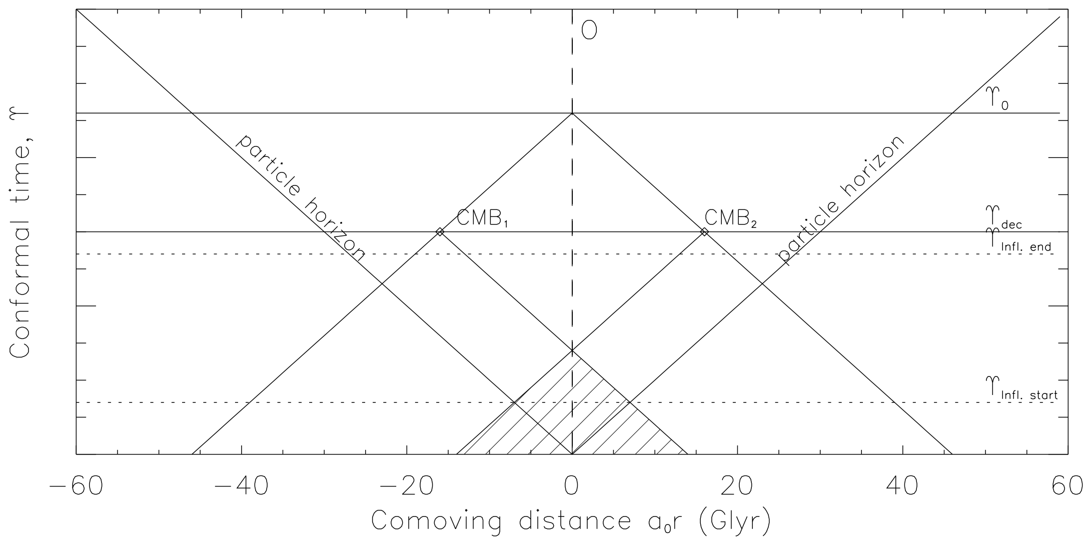

Figure 2.2: Spacetime diagram for a conformally flat universe with an inflationary epoch, starting at $\tau_{\mathrm{Infl.start}}$ and ending at $\tau_{\mathrm{Infl.end}}$.

end of inflation, it rather has to start close to its beginning.

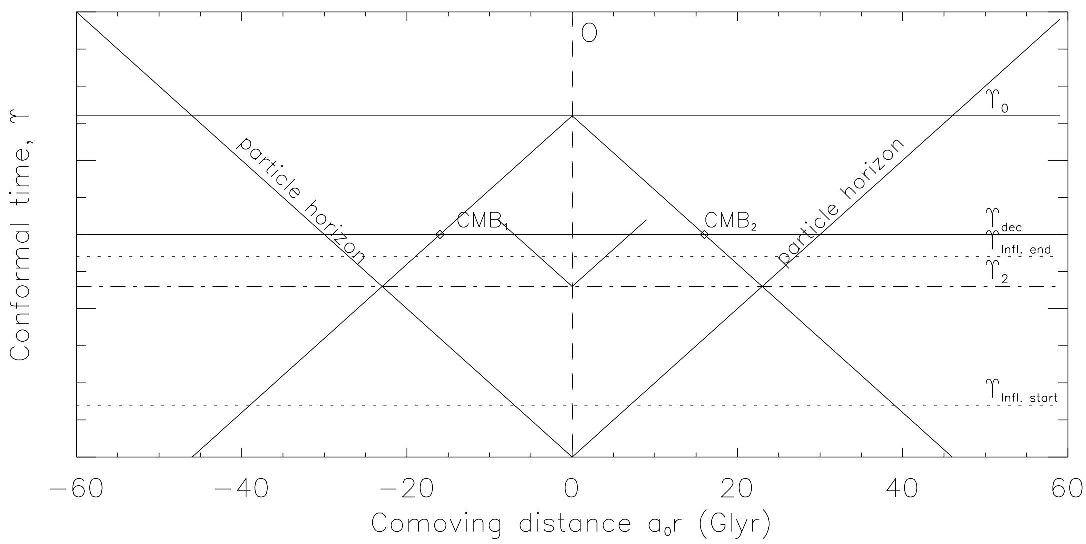

Figure 2.3: Spacetime diagram for a conformally flat universe with an inflationary epoch. τ_2 indicates when the inflaton starts to decay.

In fig. 2.3 we have selected the observer O at $r = 0$ to be inside the region of the inflating patch that is actually decaying. If the decay starts at τ_2, that is, close to the end of inflation, and occurs in a small neighborhood around us, then there is no way CMB_1 and CMB_2 can get affected by it at the time of decoupling.

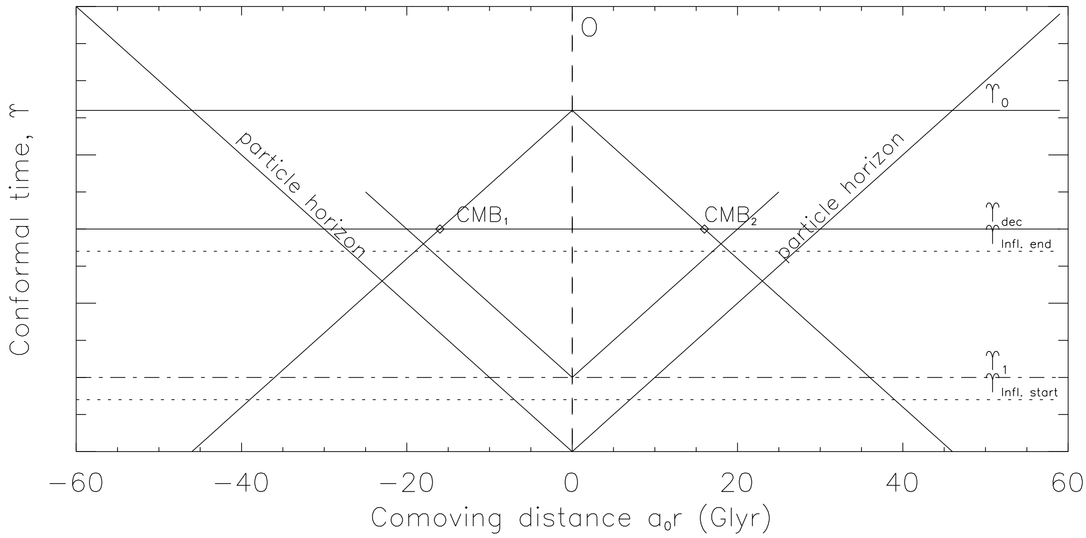

Figure 2.4: Spacetime diagram for a conformally flat universe with an inflationary epoch. τ_1 indicates when the inflaton starts to decay.

In the τ_1 case however (see fig. 2.4) the two CMB events are by far causally connected to the decaying region. This is why the authors of [12] claim that the decay of the inflaton must start soon, or be present all along the inflationary epoch. Otherwise the last scattering surface would be populated by both decayed and non-decayed regions, being therefore not homogeneous.

We think, however, that following their line of reasoning all we can say is that the time at which the decay starts and the size of the initial decaying region are not independent quantities if we want the full CMB radiation to be causally produced.

As you can see in fig. 2.5, in order to have CMB_1 and CMB_2 causally connected to the decaying region surrounding the observer O, the decay of the inflaton must not occur later than $\tau_{\max}$: otherwise CMB_1 and CMB_2 will not be causally connected to the decaying region around O, as in fig. 2.3 above.

However, when you consider the possible sizes of the decaying region, r_d, you find that they also limit the time at which the decay has to start (see figs. 2.6, 2.7): for decays starting at $\tau_{\max}$ or later, CMB_1 and CMB_2 are causally connected to the decaying region if its size exceeds

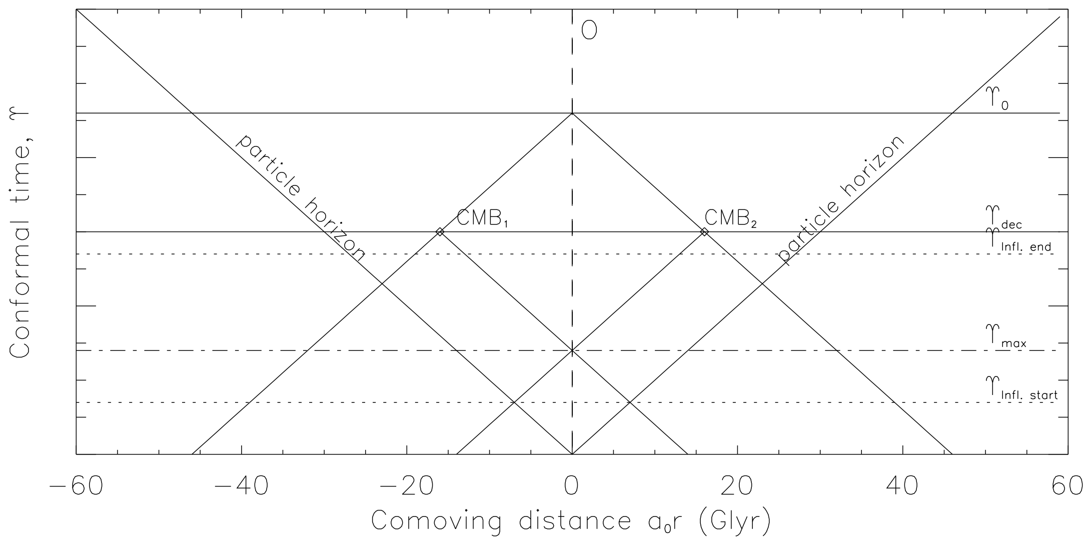

Figure 2.5: Spacetime diagram for a conformally flat universe with an inflationary epoch. $\tau_{\max}$ indicates the latest (see text), after the onset of inflation, the decay of the inflaton has to start.

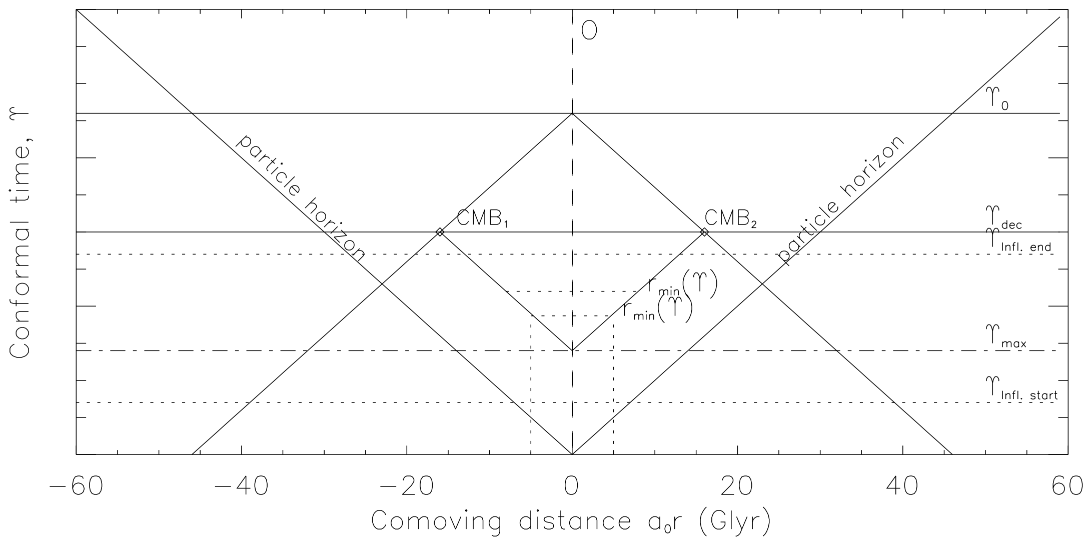

Figure 2.6: Spacetime diagram for a conformally flat universe with an inflationary epoch. If the decay of the inflaton starts after $\tau_{\max}$, the decaying region around O has to be at least of the size of

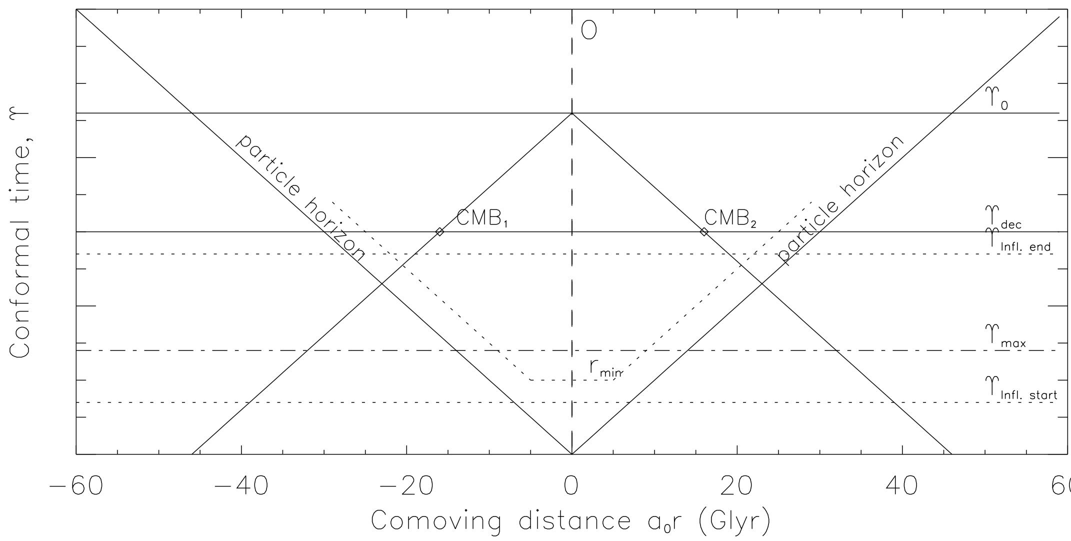

Figure 2.7: Spacetime diagram for a conformally flat universe with an inflationary epoch. If the decay of the inflaton starts before $\tau_{\max}$, any size of the decaying region around O will be causally connected to both CMB_1 and CMB_2.

$r_{min}(\tau)$, if $r_d \geq r_{min}(\tau)$; if the decay starts early, before $\tau_{\max}$, any size (of course smaller than the particle horizon) will do (fig. 2.7).

So we find that the decay of the inflaton does not necessarily have to happen all along the inflationary epoch: if the decaying region is big enough, it can happen well after inflation starts, and still an homogeneous last scattering surface is produced.

Obviously, the trivial solution to this would be having the whole inflationary patch decaying at once, in which case no causal problem would arise whatsoever. This is, however, very unlikely [20].

2.2 The event horizon during inflation

The reason why inflation produces a set of causally disconnected regions within one pocket universe, even starting from a causally connected patch, is the following: in a spatially flat exponentially expanding region, the proper size of the event horizon, D_{eh}, is constant,

$$r_{eh}(t) = c \int_t^{+\infty} \frac{dt'}{a(t')} = c \int_t^{+\infty} \frac{dt'}{a_0 e^{H(t'-t)}} = \frac{c}{a(t)H} \,, \qquad D_{eh}(t) = a(t)\; r_{eh}(t) = \frac{c}{H} \,, \tag{2.2}$$

with H constant during inflation. Therefore, each event of the inflationary spacetime will only ever be causally connected to events within this constant radius, and this for the whole duration of the inflationary epoch. However, due to the expansion, comoving structures get bigger than this, overcome the event horizon or *leave the horizon* as is usually said [30], and this way become causally disconnected.

Since, during inflation, the event horizon has the same size as the Hubble sphere (see section 1.8), we will call these regions Hubble regions, or Hubble volumes.

By the time inflation ends, after n e-folds, the initial patch has grown e^n in proper size and is composed of many causally disconnected Hubble regions. A very simple estimation [4] can give us an idea of how many of them: consider initially a Hubble region and let one Hubble time pass ($\Delta t = H^{-1}$). The region has grown by a factor of e, a factor of $e^3 \sim 20$ in volume, so after one e-fold, from a unique initial Hubble region we come up with 20 independent of them. For the standard estimation of 60 e-folds until the end of inflation we would have $\sim 20^{60} \sim 10^{78}$ non-connected Hubble volumes.

The causal isolation of each Hubble volume preserves the inflationary conditions within, allowing the full inflationary process to proceed once it has started, and independently of whatever may be happening outside [32]. This behaviour is a generalization of the "no-hair" theorems, related to black holes, to the case of de Sitter spacetimes [38], [39].

This isolation is a crucial ingredient for the inflationary expansion: any inhomogeneity inside the Hubble region will have left the horizon within a time of order the Hubble time, or one e-fold. This is how inflation actually produces the homogeneization within each Hubble volume, and for all the Hubble volumes it produces.

Also, the fact that the evolution of the field proceeds independently within such domains allows for the small inhomogeneities that will be the seeds for the structure formation in the subsequent evolution [32].

Starting from an initial homogeneous inflationary patch is clearly necessary in this context; otherwise the inhomogeneities can hardly be small enough as to preserve the 10^{-5} level of homogeneity of the last scattering surface. Furthermore, if the initial inflationary patch is not homogeneous, there is also no way to explain how the independent Hubble volumes can evolve in the same way.

2.3 Why inflation is not eternal in the past

Borde, Guth and Vilenkin (2003) [23] (hereafter BGV2003) addressed the question whether inflation is eternal in the past. When realizing that the inflationary process is eternal in the future once it starts, they asked themselves whether it could also be eternal in past directions.

There was another motivation for this work: the weak energy condition (w.e.c.) is very often violated in inflationary models due to quantum effects [37], in particular whenever the fluctuations result in an increase of the Hubble parameter, $dH/dt > 0$.

The weak energy condition,

$$T_{\mu\nu}\xi^\mu\xi^\nu \geq 0\,, \tag{2.3}$$

with ξ^μ any timelike vector, states basically that any timelike observer always measures positive energy densities. This is one of the conditions in the singularity theorems [26] that eventually lead spacetimes to be singular, so the question now was whether inflationary spacetimes, that do not respect it in general, are therefore regular. Here we mean by singular spacetimes that they contain incomplete timelike or null geodesics.

In order to answer the question, BGV2003 assume another condition for the inflationary spacetime: that the average of the Hubble parameter along past directed trayectories is never negative,

$$H_{av} \equiv \frac{1}{\lambda_f - \lambda_i}\int_{\lambda_i}^{\lambda_f} H(\lambda)d\lambda > 0\,, \tag{2.4}$$

where λ_f, λ_i are the values of the affine parameter in the past of the null or timelike geodesic along which we are calculating the average. This condition only excludes the possibility that the spacetime we are considering contracts significantly in this past epoch. The authors call this condition the "averaged expansion condition".

The goal of the authors of [23] is to see whether this condition implies incompleteness for any null or timelike geodesic.

We have reproduced all the calculations of the paper, first considering a region of the inflationary patch smaller than a Hubble volume, $(H^{-1})^3$, that is therefore well described with a flat, homogeneous and isotropic Robertson-Walker (RW) metric, $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$, and afterwards we have done the generalization for any general metric, $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$.

For each case we had to determine the behaviour of the affine parameter, whether it is bounded or not, for both a null geodesic and a timelike non-comoving geodesic.

For the null geodesics in the RW metric:

$$\int_{\lambda_i}^{\lambda_f} H(\lambda)d\lambda = \int_{t_i}^{t_f} \frac{\dot{a}(t)}{a(t)}dt = \int_{a(t_i)}^{a(t_f)} \frac{da}{a(t)} = \frac{a(t_f) - a(t_i)}{a(t_f)} \leq 1\,, \tag{2.5}$$

since $H = \frac{\dot{a}(t)}{a(t)}$ and $a(t_f) > a(t_i)$. Therefore,

$$0 < H_{av} = \frac{1}{\lambda_f - \lambda_i} \int_{\lambda_i}^{\lambda_f} H(\lambda) d\lambda \leq \frac{1}{\lambda_f - \lambda_i} \,. \tag{2.6}$$

So,

$$\lambda_f - \lambda_i \leq \frac{1}{H_{av}} \,, \tag{2.7}$$

and the value of λ_f is bounded.

For the non-comoving timelike geodesics :

$$\int_{\tau_i}^{\tau_f} H(\tau) d\tau = \int_{\tau_i}^{\tau_f} \frac{\dot{a}(\tau)}{a(\tau)} d\tau = \int_{t_i}^{t_f} \frac{m}{E} \frac{\dot{a}(t)}{a(t)} dt = \int_{a(t_i)}^{a(t_f)} \frac{m da}{\sqrt{m^2 a^2 + p_f^2 a_f^2}} \,, \tag{2.8}$$

where τ is the affine parameter of the timelike geodesic and we have used $P^0 = E = mdt/d\tau$ and $E^2 = m^2 + p^2$. The integral 2.8 has the following solution:

$$\int_{\tau_i}^{\tau_f} H(\tau) d\tau = \ln\left(\frac{m}{p_f} + \frac{E_f}{p_f}\right) - \ln\left[\frac{m a_i}{p_f a_f} + \frac{\sqrt{m^2 a_i^2 + p_f^2 a_f^2}}{p_f a_f}\right] \,. \tag{2.9}$$

The second term can be written in the form,

$$\ln\left[\frac{m a_i}{p_f a_f} + \frac{\sqrt{m^2 a_i^2 + p_f^2 a_f^2}}{p_f a_f}\right] = \ln\left[\frac{m a_i}{p_f a_f} + \sqrt{1 + \frac{m^2 a_i^2}{p_f^2 a_f^2}}\right] \,, \tag{2.10}$$

that is always ≥ 0, so

$$\int_{\tau_i}^{\tau_f} H(\tau) d\tau \leq \ln\left(\frac{m}{p_f} + \frac{E_f}{p_f}\right) \,, \tag{2.11}$$

is again bounded and therefore the timelike geodesic is incomplete.

A crucial question when generalizing this result to any spacetime is what expression for the Hubble parameter to use. The usual $H = \dot{a}/a$ is not general enough since it is specifically defined for RW metrics, so what BGV2003 do at this point is the following: they first consider a congruence of comoving timelike geodesics defined in a simple Minkowski spacetime and find a suitable expression for H in this context (see fig. 2.8).

The vector field U is the tangent vector field of the congruence as measured by the observer, O, and Δr is its infinitesimal path when moving from its intersection with one of the comoving test particles to the following. The observer measures U_1 and U_2 velocities for them.

Clearly, O will measure expansion when the two velocities are different, in particular they will see the universe expanding (or contracting) when a non-zero component of $\Delta U = U_2 - U_1$

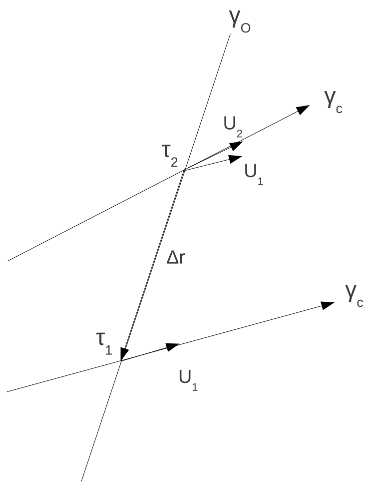

Figure 2.8: Congruence of comoving timelike geodesics, γ_c, intersected by an observer O, with geodesic γ_O, in Minkowski spacetime.

exists along its own geodesic: $\Delta U \cdot \Delta r \neq 0$. If Δr is defined from the second intersection towards the first as in fig. 2.8, then H has to be defined as follows:

$$H = \frac{-\Delta U^{\mu} \Delta r_{\mu}}{|\Delta r|^2}, \tag{2.12}$$

that is dimensionally correct. From this definition of H we can find the general expression for any metric. Let us do it step by step.

The tangent vector of the geodesic of the observer, γ_O, is V^{μ} (see fig. 2.9), and Δr^{μ} is therefore,

$$\Delta r^{\mu} = -V^{\mu} \Delta\lambda, \tag{2.13}$$

with $\Delta\lambda$ the affine parameter of γ_O. This vector has components in the direction of the comoving geodesics of the congruence:

$$-V^{\mu} \Delta\lambda U_{\mu}, \tag{2.14}$$

and components in the direction perpendicular to the comoving geodesics of the congruence:

$$-V^{\mu} \Delta\lambda + V^{\nu} \Delta\lambda U_{\nu} U^{\mu}, \tag{2.15}$$

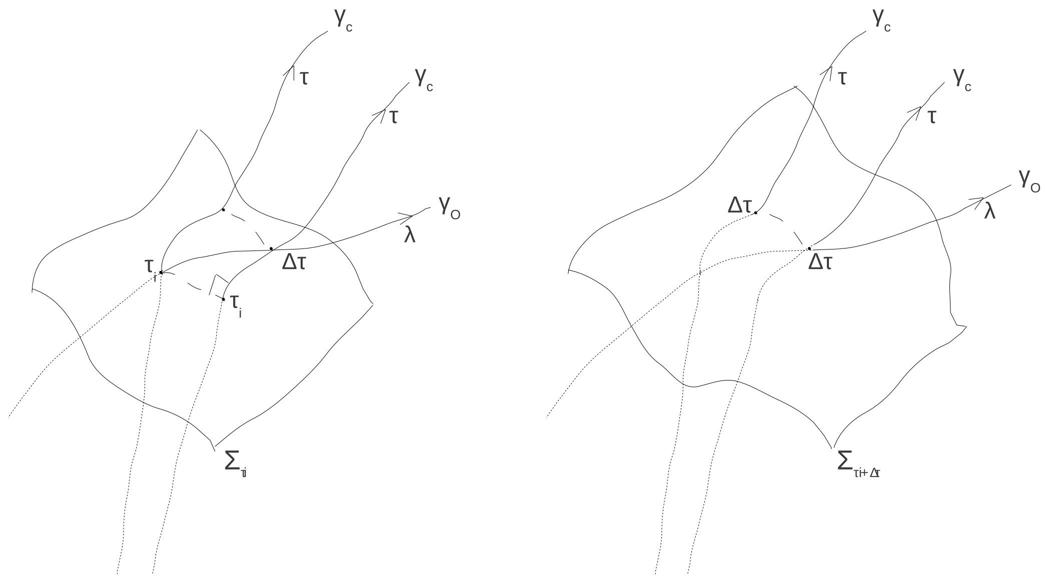

Figure 2.9: Congruence of comoving timelike geodesics, γ_c, intersected by an observer O, with geodesic γ_O, in a general spacetime $(\mathcal{M}, g_{ab})$.

so we can write

$$\Delta r^\mu = (V^\nu U_\nu U^\mu - V^\mu)\Delta\lambda = (\gamma U^\mu - V^\mu)\Delta\lambda, \tag{2.16}$$

since $\gamma \equiv V^\nu U_\nu$.

Now we go for ΔU^μ. We have to parallel transport U^μ along the geodesic of the observer and see how it changes, therefore,

$$\Delta U^\mu = \nabla_V U^\mu \Delta\lambda = V^\nu \nabla_\nu U^\mu \Delta\lambda. \tag{2.17}$$

And finally,

$$|\Delta r|^2 = \Delta r^\mu \Delta r_\mu = (k - \gamma^2)\Delta\lambda^2, \tag{2.18}$$

with $k \equiv V^\nu V_\nu$, where we have used equation 2.16.

These are all the ingredients of the general expression for the Hubble parameter:

$$H = \frac{-\Delta U^\mu \Delta r_\mu}{|\Delta r|^2} = \frac{-V^\nu \nabla_\nu U^\mu \Delta\lambda(\gamma U_\mu - V_\mu)\Delta\lambda}{(k - \gamma^2)\Delta\lambda^2} = \frac{-V_\mu V^\nu \nabla_\nu U^\mu}{(\gamma^2 - k)}, \tag{2.19}$$

where we have neglected the term $-\gamma(V^\nu \nabla_\nu U^\mu)U_\mu$ that comes out from the algebra since it is a component of $\nabla_V U^\mu$ along U^μ itself that does not contribute to the expansion (only relative displacements of the geodesics in their perpendicular directions can produce the expansion).

With some more algebra, H can be expressed:

$$H = \frac{d}{d\lambda} F(\gamma), \tag{2.20}$$

with

$$F(\gamma) = \frac{1}{\gamma}, \tag{2.21}$$

for a null observer ($k = 0$). And

$$F(\gamma) = \frac{1}{2} \ln(\frac{\gamma + 1}{\gamma - 1}), \tag{2.22}$$

for a timelike observer ($k = 1$).

We can now go back to the "averaged expansion condition" and see whether the geodesic of the observer is incomplete or not:

$$\int_{\lambda_i}^{\lambda_f} H(\lambda) d\lambda = \int_{\lambda_i}^{\lambda_f} \frac{d}{d\lambda} F(\gamma) d\lambda = F(\gamma_f) - F(\gamma_i) \leq F(\gamma_f), \tag{2.23}$$

the integral is bounded since $F(\gamma)$ is always positive. We have therefore,

$$\lambda_f - \lambda_i \leq \frac{F(\gamma_f)}{H_{av}}, \tag{2.24}$$

so the affine parameter of γ_O is bounded and therefore γ_O is past incomplete.

So, BGV2003 conclude that general inflationary spacetimes, without significant contracting phases in the past, hold incomplete null or timelike geodesics, are therefore singular in past directions, and so cannot be past eternal.

We agree however with Linde (1990) [20] when he says that cosmological singularities should be linked to all null or timelike geodesics of the spacetime being incomplete, not just some of them.

2.4 Embedding of the inflationary patch within the background spacetime

Vachaspati & Trodden (1999) [21] and Berera & Gordon (2001) [22] have studied the question of the embedding of the inflationary patch within a background non-inflationary spacetime.

They consider the embedding well behaved when an observer measures positive energy densities in the boundary of the two spacetimes, or in other words, when the boundary satisfies the weak energy condition (w.e.c.):

$$T_{\mu\nu} \xi^\mu \xi^\nu \geq 0\,, \tag{2.25}$$

with ξ^μ any timelike vector.

In order to study the behaviour of the boundary, they consider a congruence of null rays that crosses it, and study how the expansion scalar, θ (see section 1.3.1), changes as a result of this crossing. As we saw in section 1.3.1 Raychaudhuri's equation is the dynamical equation for θ, and in the case of a congruence that respects the weak energy condition it can be simplified to:

$$\frac{d\theta}{d\lambda} \leq 0\,, \tag{2.26}$$

with λ the affine parameter of the congruence.

Therefore, what the authors have to study is whether the crossing of the null congruence increases or decreases the value of θ and therefore violates (or not) the w.e.c.

Now we have to make use of the concept of **anti-trapped** region. Such regions appear in expanding spacetimes whenever the expansion is fast enough, and are characterized by the fact that both outgoing and ingoing ligth rays, emmited from a sphere centered on any observer, are divergent, have positive values for θ. In this case, the sphere is called an anti-trapped surface, and all spheres bigger than this one are also anti-trapped. In fact, in an expanding spacetime, all regions bigger than the Minimal Anti-trapped Surface (MAS) are anti-trapped.

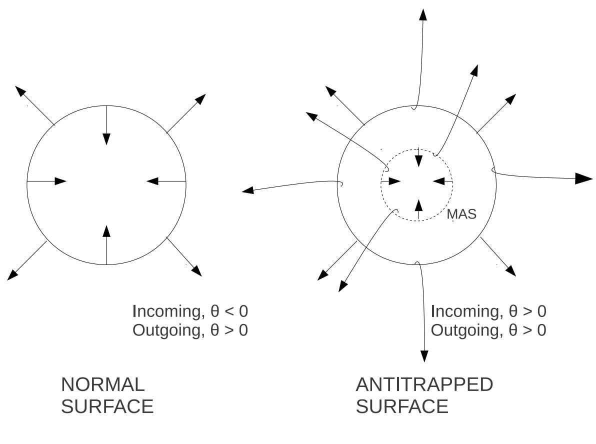

Figure 2.10: Normal and antritrapped surfaces in an expanding spacetime. MAS stands for Minimal Antitrapped Surface (see text).

Normal regions on the contrary, have convergent ingoing null geodesics and diverging outgoing ones. Therefore, since the MAS separates an inner normal region (with θ negative for ingoing null rays) from an outer anti-trapped one (with θ positive for ingoing null rays), the value of θ on the MAS has to be zero (see figure 2.10).

So the point is finding an embedding that does not violate the w.e.c, for which $d\theta/d\lambda \leq 0$, for either an ingoing null congruence or an outgoing null congruence crossing the boundary of the two spacetimes.

In the references quoted, the background non-inflationary spacetime is considered to be an expanding FRWL universe, and the inflationary spacetime is also described by a RW metric but with the scale factor increasing exponentially with time.

Under these conditions, the size of the MAS, $D_{\rm MAS}$, is:

$$D_{\rm MAS} = \begin{cases} H^{-1}\frac{1}{(1-\Omega)^{1/2}}\,\text{arcsinh}\left(\sqrt{\frac{1-\Omega}{\Omega}}\right),\ 0<\Omega<1 \\ \\ H^{-1},\ \Omega=1 \\ \\ H^{-1}\frac{1}{(\Omega-1)^{1/2}}\,\text{arcsin}\left(\sqrt{\frac{\Omega-1}{\Omega}}\right),\ 1<\Omega<2 \end{cases} \tag{2.27}$$

If the inflationary patch has developed an antitrapped surface inside and it is embedded in a normal region of the background spacetime, for incoming lightrays from the background (with $\theta<0$) into the inflating antritrapped region (with $\theta>0$) we will have $d\theta/d\lambda>0$, and, therefore, such a configuration is not a good embedding.

All the other 3 combinations, namely, a normal inflationary patch embedded in a normal background, a normal inflationary patch embedded in an anti-trapped background and an anti-trapped inflationary patch embedded in an anti-trapped background, are good embeddings because they respect the w.e.c.

Now we have to introduce in the reasoning the ingredient of causality. And here is where we differ from the authors of [21] and [22]. They say that the initial inflationary patch has to be smaller than the size of the Hubble horizon of the background spacetime in order to be causally produced. Here the confusion between Hubble sphere and event or particle horizon (as we explained in 1.8) is playing a critical role, leading, in our view, to wrong conclusions.

Anyway, let us follow their line of reasoning. We have an initial inflationary patch that is smaller than the background Hubble horizon, $D_0 < 1/H_{bg}$, and therefore is causally connected. Since for the $\Omega=1$ spacetimes of the authors of [21] the size of the MAS equals that of the Hubble horizon (see eq. 2.27), the initial inflationary patch is inside a normal region of the background spacetime.

In order for the initial inflationary patch to be stable against perturbations, its initial size,

D_0, has to be larger than the Hubble horizon, $D_0 > 1/H_{inf}$, and therefore the boundary of the inflationary patch is in an anti-trapped region of the inflationary spacetime. So we have an anti-trapped inflationary region surrounded by a normal background region, that is something that, as we have already explained, leads to the violation of the w.e.c.

If the initial inflationary patch is larger than the background Hubble horizon, the exterior will also be an anti-trapped region, in which case no violation of the w.e.c. should occur. The authors of [21], therefore, conclude that the initial inflationary patch has to have an acausal size, something that can actually invalidate the whole inflationary idea.

The work of Berera & Gordon (2001) [22] came to solve the puzzle. They generalized the calculations of the previous authors to an initial inflationary patch with any global geometry, that can subsequently evolve to the flat $\Omega = 1$ geometry with the exponential expansion.

When the initial geometry of the inflationary patch is open, $\Omega < 1$, the size of the MAS is larger than that of the Hubble horizon (see eq. 2.27), so we can have an initial inflationary patch in a normal region, larger than the inflationary Hubble horizon and therefore still stable against perturbations. If we preserve the causal origin of the patch and therefore it is initially smaller than the background Hubble horizon, we will have a normal inflationary region embedded in a normal background region, something that respects the weak energy condition provided that θ is smaller in the inflationary side of the boundary.

As inflation proceeds, $\Omega \to 1$, and the size of the inflationary patch becomes larger than $1/H_{inf}$, so we have the boundary in an anti-trapped region of the inflationary spacetime. To respect now the w.e.c., the background spacetime on the other side of the boundary also has to be an anti-trapped region, so the MAS size of the background spacetime has to be smaller than the size of the inflationary region at that time. The authors of [22], however, conclude that the size of the background MAS has to be smaller than the one of the inflationary MAS, a condition that still produces a good embedding but that is much more restrictive.

Summarizing, Berera & Gordon [22] find good embedding solutions that preserve the causal formation of the initial inflationary patch, giving a solid base to the inflationary idea in terms of causality.

We would like, however, to reproduce this work but eliminating the use of Hubble horizons, by using as criterium for the initial inflationary patch to be causally connected that it is smaller than the particle horizon of the background spacetime, rather than smaller than the Hubble sphere (the criterium incorrectly used in these works).

Chapter 3

Conclusions and future work

We have explored the idea of using causality considerations to constrain inflationary theories and to check for their internal consistency.

Ellis & Stoeger (1988) [12] did this, finding that causality actually constrains the time when the inflaton field has to start decaying, that at the moment was considered to be very close to the end of inflation. After their analysis it has been considered to start soon after the onset of inflation, or from its very beginning.

Other authors that have analysed the internal consistency of the inflationary scenario in terms of causality, although somewhat indirectly, are Vachaspati & Trodden (1999) [21]. They studied the embedding of the inflationary patch within its background non-inflationary spacetime, requiring their boundary to be energetically well behaved, in the sense that positive energy densities would be measured by observers crossing it.

Their work analyses only the flat case, $\Omega = 1$, for both the inflationary and non-inflationary expanding background, and they surprisingly find that the only way to have a good embedding under these conditions is to start with an inflationary patch that is bigger than the causal border of the background spacetime at that moment.

This result is shocking because it can invalidate the whole inflationary idea: if the exponential expansion solves the causal problem related to the homogeneity (and isotropy) of the CMB radiation (the horizon problem) but the initial inflationary region still has to be acausal in order to do it, the internal consistency of the inflationary scenario is in doubt.

After studying in detail this work, we disagree with their criterium to consider a region causally connected: they use the Hubble horizon as the causal border of the universe at a given time of its evolution, while it is the particle horizon what should be used instead.

As we explained in section 1.8, following the work of Davis & Lineweaver (2004) [30], the term Hubble horizon leads to confusion and refers to two, in general, different concepts. One is the Hubble sphere, which is the surface at which recession velocities of comoving objects equals the speed of light. And the other is the event horizon, that is really a causal horizon, in fact it is the future causal horizon of every event of the considered spacetime. In flat inflationary spacetimes the event horizon and the Hubble sphere coincide, but for other expansion histories, not exponential, this is not neccesarily true. Furthermore, what dictates whether or not certain region of the spacetime can have been causally formed is the size of the particle horizon, not that of the event horizon.

An early response to the work of Vachaspati & Trodden (1999) [21] is the paper by Berera & Gordon (2001) [22] that solves the causal puzzle but not the Hubble horizon confusion. These authors improve the previous analysis by allowing the initial inflationary patch to be spatially open, with $\Omega < 1$, and find this way embeddings energetically well behaved for initial sizes smaller than the Hubble horizon, therefore causally connected.

We would like to addres this question in the near future, improving the latter analysis with the more precise definition of causally connected region we have discussed above.

Regarding the constraint causality can impose on how long after the onset of inflation the inflaton has to start decaying, we have found that there exists an upper limit, we have called τ_{max}, above which the decay can start at any time provided the size of the decaying region is big enough (while still smaller than the particle horizon). Quantifying this is another question we want to address soon as the natural continuation of this work.

Another issue we have tackled regards the non-past eternal nature of the inflationary process. By construction, inflation is eternal in future directions once it starts. Borde, Guth and Vilenkin [23] asked themselves whether it could also be eternal in past directions. We have reproduced here their calculations, that show that inflationary spacetimes of any metric, with positive average expansion rates, are singular, with incomplete past causal geodesics, and therefore not past eternal.

We would also like to generalize many basic definitions regarding spacetime horizons making use of the general form for the Hubble expansion rate, H, that the previous authors have developed in order to prove their theorem.

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