Monopoles, instantons and non-Abelian black holes

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Based on 1503.01044 1512.07131 1605.00005 and work in preparation.

Work done in collaboration with *P.F. Ramírez* (IFT UAM/CSIC, Madrid) and *P. Meessen* (U. Oviedo)

Plan of the Talk:

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The timelike supersymmetric solutions of $\mathcal{N} = 1, d = 5$ SEYM theories were classified in 0705.2567 (earlier) but no non-Abelian black-hole solutions were constructed until very recently (1512.07131).

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We are going to present these equations and some relevant solutions.

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$$\Phi_{\Lambda}\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi^{\Lambda} - \Phi^{\Lambda}\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi_{\Lambda} = 0\,.$$

In general there will be a Abelian sector (λ) and a non-Abelian sector (A) which will always be SU(2) in this talk:

$$\begin{aligned}
\tfrac{1}{2}\varepsilon_{\underline{rsw}}\breve{F}^{\lambda}{}_{\underline{sw}} - \partial_{\underline{r}}\Phi^{\lambda} &= 0\,, \quad \Rightarrow \quad \partial_{\underline{r}}\partial_{\underline{r}}\Phi^{\lambda} = 0\,, \\
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\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi_{A} - g^{2}\left(\Phi^{B}\Phi^{B}\Phi_{A} - \Phi^{A}\Phi^{B}\Phi_{B}\right) &= 0\,, \\
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The solutions of the Abelian sector are completely determined by a choice of harmonic functions $\Phi^{\lambda}, \Phi_{\lambda}$ in $\mathbb{E}^3$. What happens in the non-Abelian sector?

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The last set of equation mixing Φ^{A}, Φ_{A}, $\breve{A}_{\underline{r}}$ is automatically solved except at the singularities, where one has to impose conditions on the integration constant (Denef,Bates.)

3 – Solutions to the SU(2) Bogomol'nyi equations: Protogenov's

All the spherically-symmetric configurations Φ^A, $\breve{A}_{\underline{r}}$ can be brought to the form (*hedgehog ansatz*)

$$\Phi^A = -\delta^A{}_r f(r) x^r , \qquad \breve{A}^A{}_{\underline{r}} = -\varepsilon^A{}_{rs} x^s h(r),$$

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The Bogomol'nyi equations become an system of ODFs for $f(r)$ and $h(r)$

$$\begin{cases} r\partial_r h + 2h + f(1 + gr^2 h) & = & 0\,, \\ r\partial_r (h - f) - gr^2 h(h - f) & = & 0\,. \end{cases}$$

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- A 2-parameter family (μ and s, a.k.a. *Protogenov "hair"*)

$$f_{\mu,s} = \frac{1}{gr^2}\left[1 - \mu r \coth\left(\mu r + s\right)\right], \qquad h_{\mu,s} = \frac{1}{gr^2}\left[1 - \frac{\mu r}{\sinh\left(\mu r + s\right)}\right],$$

$s = 0$ is the 't Hooft-Polyakov monopole in the BPS limit and $s = \infty$ the Wu-Yang SU(2) monopole (plus a constant).

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- A 1-parameter (λ) family of *coloured* monopoles

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The *coloured* monopoles are very interesting solutions: their charge is screened at infinity and they can be generalized to multicenter solutions.

4 – Solutions to the SU(2) Bogomol'nyi equations: Ramírez's

Recently (1608.01330), Ramírez has shown that the SU(2) Bogomol'nyi equations are solved by

$$\Phi^{A} = \delta^{A\underline{r}} \frac{1}{gP} \partial_{\underline{r}} P \,, \qquad \breve{A}^{A}{}_{\underline{r}} = \varepsilon^{A}{}_{rs} \frac{1}{gP} \partial_{\underline{s}} P \,,$$

where P is any real function satisfying

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For just one pole, this is the coloured monopole with $\lambda^2 = P_0/P_1$.
Many poles: many coloured monopoles in equilibrium.
All the coefficients of the poles must have the same sign.

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The last equation is solved quite non-trivially everywhere: no constraints on the integration constants!

Now, given a solution

$$\Phi^{\lambda}, \Phi_{\lambda}, \Phi^{A}, \Phi_{A}, \breve{A}_{\underline{r}}$$

to the equations, we construct **supergravity solutions**

AS FOLLOWS:

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The last is the most apropriate for us because

$$\begin{pmatrix} \mathcal{I}^\Lambda \\ \mathcal{I}_\Lambda \end{pmatrix} = -\sqrt{2} \begin{pmatrix} \Phi^\Lambda \\ \Phi_\Lambda \end{pmatrix},$$

and the $\breve{A}^\Lambda_{\underline{r}}$ are the corresponding part of the $\mathcal{N} = 2, d = 4$ supergravity vector fields.

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- The metric has the form

$$ds^2 = \mathsf{W}^{-1}(dt+\omega)^2 - \mathsf{W}dx^r dx^r ,$$

where the 1-form $\omega = \omega_{\underline{r}<} dx^r$ on $\mathbb{R}^3$ is found by solving the equation

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Let us see some choices with good properties (we focus on the metric only for the sake of simplicity).

Global monopole: $H = 1+$ BPS 't Hooft-Polyakov monopole

$$ds^2 = \mathsf{W}^{-1}dt^2 - \mathsf{W}(dr^2 + r^2 d\Omega^2_{(2)})\,, \quad \text{where} \quad \mathsf{W} = 1 - \frac{1}{g^2 r^2}\left[1 - \mu r \coth(\mu r)\right]^2 \,.$$

Globally regular. Mass but no horizon nor entropy. (BPS 't Hooft-Polyakov monopoles always do this when combined with other fields).

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***Dumbbell* solution:** $H = p^0/r+$ coloured monopole (in $d = 6$, Cano, Ortín & Santoli (2016)):

$$ds^2 = \mathsf{W}^{-1}dt^2 - \mathsf{W}(dr^2 + r^2 d\Omega^2_{(2)})\,, \quad \text{where} \quad \mathsf{W} = \frac{(p^0)^2}{r^2} - \frac{1}{g^2 r^2}\left[\frac{1}{1 + \lambda^2 r}\right]^2 .$$

Flows fom one $AdS_2 \times S^2$ to another $AdS_2 \times S^2$ of different radius!

Multi-coloured black holes: $H = 1 + \sum_\alpha p^0_\alpha / |\vec{x} - \vec{x}_\alpha| +$ coloured monopoles
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These are the simplest, but more general solutions are possible (dyonic, with objects of different types, black hedgehogs, etc.).

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We are interested in a special class of solutions that can be described in terms of functions M, H, Φ^I, L_I, which are related to the building blocks Φ^Λ, Φ_Λ, $\Lambda = 0,1,\cdots,n_{V5}+1$ by

$$\Phi^I = \Phi^{I+1}\,, \qquad L_I = -\frac{2\sqrt{2}}{3}\Phi_{I+1}\,, \qquad H = -2\sqrt{2}\Phi^0\,, \qquad M = +\sqrt{2}\Phi_0\,.I = 1,\cdots,n_{V5}\,.$$

The 5-dimensional metric has the form

$$ds^2 = (\mathsf{W}/2)^{-4/3}(dt + \hat{\omega})^2 - (\mathsf{W}/2)^{2/3}\left[H^{-1}(dz + \chi)^2 + H dx^r dx^r\right] ,$$

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and can be reconstructed from the above functions as follows:

$$\begin{aligned} d\chi &= \star_3 dH\,, \\ dH_I &= L_I + 8C_{IJK}\Phi^J\Phi^K/H\,, \\ \hat{\omega} &= \omega_5(dz + \chi) + \omega\,. \end{aligned}$$

where ω is the same one would find for the 4-dimensional solution.

Simplest HK metric: $H = 1, \omega = 0$, which is $\mathbb{R}^4$. The uplifted monopoles will have a translational invariance and the metric a translational isometry:

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We may obtain black holes, but beware of the singularities!!.

9 – A simple example with gauge group SU(2)

It is given by $C_{0\Lambda\Sigma} = \frac{1}{3!}\eta_{\Lambda\Sigma}$ $\Lambda\Sigma = 1, x$ $x, y = A+1$ or by

$$\mathsf{W} = \left\{\tfrac{27}{2} H_0 \eta^{\Lambda\Sigma} H_\Lambda H_\Sigma\right\}^{1/2} ,$$

which gives

$$(\mathsf{W}/2)^{2/3} = H^{-1}\left\{\tfrac{1}{4}\left(6HL_0 + 8\eta_{xy}\Phi^x\Phi^y\right)\left[9H^2\eta^{xy}L_xL_y + 48H\Phi^0 L_x\Phi^x + 64(\Phi^0)^2\eta_{xy}\Phi^x\Phi^y\right]\right\}^{1/3} .$$

The simplest solution has just H, L_0, L_1, Φ^{A+1}

$$(\mathsf{W}/2)^{2/3} = \left\{\tfrac{27}{2}\left(L_0 - \tfrac{4}{3}\Phi^{A+1}\Phi^{A+1}\right)(L_1)^2\right\}^{1/3} .$$

and it is just a D1D5W black hole with a non-Abelian contribution which has to be the BPST instanton for one center (more centers are under investigation)

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10 – Conclusions

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- They present interesting new features that, for the first time, can be studied analytically.
- Explaining these entropies from a mircorscopic point of view presents a new challenge to superstring theory.
- More general non-Abelian solutions can be obtained: black rings (Ortín, Ramírez, 1605.00005), microstate geometries (Ramírez, 1608.01330) , and non-extremal black holes (work in progress).

THANKS!

11 – Instantons Vs. Monopoles

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where (unhatted $\Rightarrow \mathbb{E}^3$)

$$dH = \star d\chi\,, \quad \Rightarrow \quad d \star dH = 0\,, \quad \text{in} \quad \mathbb{R}^3\,.$$

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For any gauge group G, let $\hat{A}$ be a gauge field whose field strength $\hat{F}$ is selfdual $\hat{\star}\hat{F} = +\hat{F}$ in the above HK metric (**orientation!**).

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where (unhatted $\Rightarrow \mathbb{E}^3$)

$$dH = \star d\chi\,, \quad \Rightarrow \quad d \star dH = 0\,, \quad \text{in} \quad \mathbb{R}^3\,.$$

For any gauge group G, let $\hat{A}$ be a gauge field whose field strength $\hat{F}$ is selfdual $\hat{\star}\hat{F} = +\hat{F}$ in the above HK metric (**orientation!**).

Then, the 3-dimensional gauge and Higgs fields A and Φ defined by

$$\Phi \equiv -H\hat{A}_z\,,$$

$$A_{\underline{r}} \equiv \hat{A}_{\underline{r}} - \chi_{\underline{r}}\hat{A}_z\,,$$

satisfy the Bogomol'nyi equation in $\mathbb{E}^3$ $\mathfrak{D}_{\underline{r}}\Phi = \frac{1}{2}\epsilon_{\underline{rst}}F_{\underline{st}}$.