Black Holes in Supergravity and Superstring Theories

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General review + work done in collaboration with P. Galli, (U. Valencia), P. Meessen, (U. Oviedo), M. Hübscher, J. Perz, C.S. Shahbazi, S. Vaulà (IFT-UAM/CSIC, Madrid)

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Plan of the Talk:

1 – Introduction

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In this talk we are going to review general properties of the solutions of Supergravity and some general families of black-hole solutions. We will restrict our attention to **static** black holes in **4 dimensions** and we will focus specially on $N = 2$ Supergravity .

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This is a generalization of the concept of isometry, an infinitesimal g.c.t. generated by a $\xi^\mu(x)$ that leaves the metric $g_{\mu\nu}$ invariant

$$\delta_\xi g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)} = 0\,. \quad (\textit{Killing (vector) equation})$$

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The mass of the static solutions of Einstein-Maxwell theory satisfies the same BPS bound as the states of $N = 2$ Supergravity (Gibbons & Hull (1982)):

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When the BPS bound is saturated, the solution turns out to be BPS and the Reissner-Nordström black hole becomes extremal.

In more general $N = 2$ Supergravity theories (more scalars, Z^i, and more vectors A^Λ and electric q_Λ and magnetic p^Λ charges) with the central charge $\mathcal{Z}_\infty \equiv \mathcal{Z}(Z^i{}_\infty, q, p)$

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☞ There are extremal black holes which are **not** BPS (Khuri & Ortín, (1997)). The extremality bound cannot be just $r_0{}^2 = M^2 - |\mathcal{Z}_\infty|^2 \geq 0$. Do we need to find all the extremal and non-extremal solutions?

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- ☞ The maximally supersymmetric solutions (not always maximally symmetric) can be interpreted as vacua. Configurations preserving less supersymmetry spatially interpolate between them.
- ☞ Last, but not least, BPS configurations are simple, depend on very few independent functions and (the fields) satisfy 1^{st} order (*flow*) differential equations that have *attractor points* for the scalar fields.

We would like to know which of these properties are shared by the extremal but non-supersymmetric black hole solutions.

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(1) Algebraic approach
- (Ferrara, Gibbons & Kallosh, (1997)) (general formalism)
- Ceresole & Dall'Agata (2007) ("fake" superpotentials)

(2) Explicit solutions

Supersymmetric (*i.e.* extremal) :
- Tod (1983) (pure $N = 2$)
- Behrndt, Luest & Sabra (1997)(general $N = 2$)
- Caldarelli & Klemm (2003) (Abelian – gauged $N = 2$)
- Huebscher, Meessen, O. & Vaula (2007), Meessen, (2008) (non – Abelian – gauged $N = 2$)
- Meessen, O. & Vaula (2010) (all $N \geq 2$)

Non – extremal :
- Cvetic & Youm (1996)
- O. (1996)
- Kastor & Win (1996)
- Mohaupt & Vaughan (2010) (general Ansatz $d = 5$)
- Galli, O., Perz & Shahbazi (2011) (general Ansatz $d = 4$)

4 – Algebraic (FGK) approach

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They also considered the general metric for any static non-extremal black hole

$$ds^2 = e^{2U(\tau)}dt^2 - e^{-2U(\tau)}\left[\frac{{r_0}^4}{\sinh^4 r_0\tau}d\tau^2 + \frac{{r_0}^2}{\sinh^2 r_0\tau}d\Omega^2_{(2)}\right]\ .$$

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For the Schwarzschild black hole $r_0 = 2M$ and $U = e^{r_0\tau}$.

The general system reduces to an effective mechanical system with variables $U(\tau), \phi^i(\tau)$:

$$I_{\text{eff}}[U, \phi^i] = \int d\tau \left\{ (U')^2 + \frac{1}{2}\mathcal{G}_{ij}\phi^{i\,\prime}\phi^{j\,\prime} - e^{2U}V_{\text{bh}} + r_0{}^2 \right\} ,$$

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where FGK defined the black-hole potential

$$-V_{\rm bh}(\phi, q, p) \equiv -\frac{1}{2}(p^\Lambda \quad q_\Lambda) \begin{pmatrix} (\mathfrak{I} + \mathfrak{R}\mathfrak{I}^{-1}\mathfrak{R})_{\Lambda\Sigma} & -(\mathfrak{R}\mathfrak{I}^{-1})_\Lambda{}^\Sigma \\ -(\mathfrak{I}^{-1}\mathfrak{R})^\Lambda{}_\Sigma & (\mathfrak{I}^{-1})^{\Lambda\Sigma} \end{pmatrix} \begin{pmatrix} p^\Sigma \\ q_\Sigma \end{pmatrix} ,$$

where

$$\mathfrak{R}_{\Lambda\Sigma} \equiv \Re\mathrm{e}\mathcal{N}_{\Lambda\Sigma}(\phi)) , \qquad \mathfrak{I}_{\Lambda\Sigma} \equiv \Im\mathrm{m}\mathcal{N}_{\Lambda\Sigma}(\phi)) , \qquad (\mathfrak{I}^{-1})^{\Lambda\Sigma}\mathfrak{I}_{\Sigma\Gamma} = \delta^\Lambda{}_\Gamma .$$

The general system reduces to an effective mechanical system with variables $U(\tau), \phi^i(\tau)$:

$$I_{\rm eff}[U, \phi^i] = \int d\tau \left\{ (U')^2 + \frac{1}{2}\mathcal{G}_{ij}\phi^{i\,\prime}\phi^{j\,\prime} - e^{2U}V_{\rm bh} + {r_0}^2 \right\} ,$$

where FGK defined the black-hole potential

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Finding a black hole with charges p, q is equivalent to solving the above system for $U(\tau), \phi^i(\tau)$.

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Each critical point yields a possible extremal black-hole solution and an $AdS_2 \times S^2$ geometry. One can go a long way with the attractor only, ignoring the full explicit solution.

In the general case one can prove the following extremality bound:

$$r_0{}^2 = M^2 + \tfrac{1}{2}\mathcal{G}_{ij}(\phi_\infty)\Sigma^i\Sigma^j + V_{\text{bh}}(\phi_\infty, q, p)\,, \geq 0\,,$$

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$$\begin{aligned} U &\sim 1 + M\tau\,, \\ \phi^i &\sim \phi^i_\infty - \Sigma^i\tau\,. \end{aligned}$$

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We need to find the complete explicit solution in the non-extremal case.

Whenever we can write $-\left[e^{2U} V_{\rm bh} - {r_0}^2\right] = (\partial_U Y)^2 + 2\,\mathcal{G}^{ij}\partial_i Y \partial_j Y$ for some *(generalized) superpotential* $Y(U, \phi^i, p, q, r_0)$, we can rewrite the effective action as

$$I_{\rm eff}[U, \phi^i] = \int d\tau \left\{(U' - \partial_U Y)^2 + \tfrac{1}{2}\mathcal{G}_{ij}(\phi^{i\,\prime} - 2\,\mathcal{G}^{ik}\partial_k Y)(\phi^{j\,\prime} - 2\,\mathcal{G}^{jl}\partial_l Y) + 2Y'\right\} .$$

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The action is minimized by configurations satisfying the first-order gradient flow equations (Miller, Schalm & Weinberg (2007), Janssen, Smyth, Van Riet & Vercnocke (2008), Perz, Smyth, Van Riet & Vercnocke (2008))

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A generalized superpotential $Y(U, \phi^i, p, q, r_0)$ exists in all theories whose scalar manifold (after timelike dimensional reduction) is a symmetric coset space (in particular for all $N > 2$ supergravities) (Andrianopoli, D'Auria, Orazi & Trigiante (2009), Chemissany, Fré, Rosseel, Sorin, Trigiante & Van Riet (2010)).

In the extremal case $r_0 = 0$, if there is a generalized superpotential $Y(U, \phi^i, p, q)$, it factorizes

$$Y(U, \phi^i, p, q) = e^U W(\phi^i, p, q)\,,$$

where $W(\phi^i, p, q)$ is called the *superpotential*, and the flow equations take the form (Ceresole & Dall'Agata (2007))

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The stationary values of the superpotential $\partial_i W|_{\phi_{\rm h}} = 0$ give the the entropy:

$$S = \pi |W(\phi_{\rm h}, p, q)|^2\,,$$

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$$M = |W(\phi_\infty, p, q)|\,.$$

5 – Direct construction of solutions: extremal supersymmetric

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We are going to review two examples:

- ☞ (Ungauged) $N = 2$ Supergravity coupled to vector multiplets.
- ☞ Non-Abelian gaugings of the above theory.

6 − $N = 2, d = 4$ ungauged SUGRA coupled to vector multiplets

$6-N=2, d=4$ ungauged SUGRA coupled to vector multiplets

The field content

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All vector fields are collectively denoted by $A^\Lambda{}_\mu = (A^0{}_\mu, A^i{}_\mu)$ and the complex scalars Z^i are described by constrained symplectic sections $(\mathcal{L}^\Lambda(Z, Z^*), \mathcal{M}_\Lambda(Z, Z^*))$.

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n_H Hypermultiplets $(u = 1, \cdots 4n_H \;,\quad \alpha = 1, \cdots 2n_H)$	q^u	ζ_α	$(0, 1/2)$
The supergravity multiplet	$A^0{}_\mu, e^a{}_\mu$	$\psi_{I\mu}$	$(1, 2, 3/2)$

All vector fields are collectively denoted by $A^\Lambda{}_\mu = (A^0{}_\mu, A^i{}_\mu)$ and the complex scalars Z^i are described by constrained symplectic sections $(\mathcal{L}^\Lambda(Z, Z^*), \mathcal{M}_\Lambda(Z, Z^*))$.
All fermions are represented by chiral 4-component spinors:

$$\gamma_5 \psi_{I\mu} = -\psi_{I\mu}\,, \text{etc.}$$

6 − $N = 2, d = 4$ ungauged SUGRA coupled to vector multiplets

The field content

The basic $N = 2, d = 4$ massless supermultiplets are

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Hypermultiplets can be ignored for black-hole solutions.

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Local $N=2$ supersymmetry requires the Kähler-Hodge manifold to be a special Kähler manifold, so it is the base space of a $2(n_V+1)$-dimensional vector bundle with $Sp[2(n_V+1),\mathbb{R}]$ structure group, on which we can define the constrained symplectic section

$$\mathcal{V} = \begin{pmatrix} \mathcal{L}^\Lambda(Z,Z^*) \\ \mathcal{M}_\Lambda(Z,Z^*) \end{pmatrix} .$$

$\mathcal{V}$ can be thought of as just a redundant description of the physical scalars with manifest symplectic symmetry, which also acts on the electric and magnetic charges:

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The action of the bosonic fields of the ungauged theory is

$$\begin{aligned} S &= \int d^4x \sqrt{|g|}\, \big[R + 2\mathcal{G}_{ij^*}\partial_{\mu}Z^i\partial^{\mu}Z^{*\,j^*} + 2\Im\mathrm{m}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda\,\mu\nu}F^{\Sigma}{}_{\mu\nu} \\ & \qquad -2\Re\mathrm{e}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda\,\mu\nu}\star F^{\Sigma}{}_{\mu\nu}\big]\;. \end{aligned}$$

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$$\begin{pmatrix} \mathcal{I}^{\Lambda} \\ \\ \mathcal{I}_{\Lambda} \end{pmatrix} = \begin{pmatrix} \mathcal{I}^{\Lambda}_{\infty} - \frac{1}{\sqrt{2}} p^{\Lambda} \tau \\ \\ \mathcal{I}_{\Lambda\,\infty} - \frac{1}{\sqrt{2}} q_{\Lambda} \tau \end{pmatrix}\,, \qquad \mathcal{I}^{\Lambda}_{\infty} q_{\Lambda} - \mathcal{I}_{\Lambda\infty} p^{\Lambda} = 0\,.$$

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4. The scalars Z^i are given by the quotients

$$Z^i = \frac{\mathcal{V}^i/X}{\mathcal{V}^0/X} = \frac{\mathcal{R}^i + i\mathcal{I}^i}{\mathcal{I}^0 + i\mathcal{I}^0}\,.$$

4. The metric takes the form (in FGK coordinates)

$$ds^2 = e^{2U} dt^2 - e^{-2U} \left[\frac{d\tau^2}{\tau^4} + \frac{1}{\tau^2} d\Omega^2_{(2)} \right] .$$

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One can check in the explicit solutions all the properties predicted by the algebraic approach.

In this case the solutions do not give much more information than the algebraic approach, but they are going to be used as starting point for the construction of non-extremal solutions later on.

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In this way, genuinely no-Abelian black-hole solutions have been obtained in fully analytic form (unlike Bartnik & McKinnon's). They exhibit gauge-covariant attractors (Hübscher, Meessen, O. &,Vaulà (2007), Meessen (2008)).

7 – Direct construction of solutions: non-extremal

Based on the study of several examples, the following prescription to deform the extremal supersymmetric solutions of $N = 2$ Supergravity theories has been given (Galli, O., Perz & Shahbazi (2011)):

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If the supersymmetric solution is given by

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where $U_{\rm e}$ and $Z^i_{\rm e}$ depend on harmonic functions $H(\tau) = H_\infty - q_\alpha \tau/\sqrt{2}$ given by the standard prescription for supersymmetric black holes ,

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Then, the non-extremal solution is given by

$$U(\tau) = U_{\rm e}[\hat{H}(\tau)] + r_0\tau\,, \qquad Z^i(\tau) = Z^i{}_{\rm e}[\hat{H}(\tau)]\,,$$

where where the harmonic functions H have been replaced by

$$\hat{H} = a + be^{2r_0\tau}\,,$$

and the constants a, b have to be determined by explicitly solving the e.o.m.

We are going to give an explicit example, showing that one can recover both the extremal supersymmetric and non-supersymmetric black holes of a model from the general non-extremal solution found with this prescription.

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8 – A complete example: $\overline{\mathbb{CP}}^n$ model

This model and has n scalars Z^i to which we add for convenience $Z^0 \equiv 1$, so we have

$$(Z^\Lambda) \equiv (1, Z^i)\,, \qquad (Z_\Lambda) \equiv (1, Z_i) = (1, -Z^i)\,, \qquad (\eta_{\Lambda\Sigma}) = \mathrm{diag}(+ - \cdots -)\,.$$

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The Kähler potential and metric $(SU(1,n)/SU(n))$ are

$$\mathcal{K} = -\log\left(Z^{*\Lambda} Z_\Lambda\right), \qquad \mathcal{G}_{ij^*} = -e^{\mathcal{K}}\left(\eta_{ij^*} - e^{\mathcal{K}} Z^*_i Z_{j^*}\right)\,.$$

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It is convenient to define the complex charge combinations

$$\Gamma_\Lambda \equiv q_\Lambda + \tfrac{i}{2}\eta_{\Lambda\Sigma} p^\Sigma\,.$$

The central charge $\mathcal{Z}$, its holomorphic Kähler -covariant derivative and the black-hole potential are given by

$$
\begin{aligned}
\mathcal{Z} &= e^{\mathcal{K}/2} Z^{\Lambda} \Gamma_{\Lambda}\,, \\
\mathcal{D}_i \mathcal{Z} &= e^{3\mathcal{K}/2} Z_i^{*} Z^{\Lambda} \Gamma_{\Lambda} - e^{\mathcal{K}/2} \Gamma_i\,, \\
|\tilde{\mathcal{Z}}|^2 &\equiv \mathcal{G}^{ij^*} \mathcal{D}_i \mathcal{Z} \mathcal{D}_{j^*} \mathcal{Z}^* = e^{\mathcal{K}} |Z^{\Lambda} \Gamma_{\Lambda}|^2 - \Gamma^{*\,\Lambda} \Gamma_{\Lambda}\,, \\
-V_{\rm bh} &= 2 e^{\mathcal{K}} |Z^{\Lambda} \Gamma_{\Lambda}|^2 - \Gamma^{*\,\Lambda} \Gamma_{\Lambda}\,.
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Remember that in $N = 2$ theories, in the extremal case $|\mathcal{Z}|$ plays the rôle of superpotential W. In this case $|\tilde{\mathcal{Z}}|$ will play the rôle of "fake" superpotential.

In this case we can write

$$-\left[e^{2U}V_{\rm bh} - {r_0}^2\right] = \Upsilon^2 + 4\,\mathcal{G}^{ij^*}\,\Psi_i\Psi^*_{j^*}\,,$$

where

$$\begin{aligned}\Upsilon &= \frac{e^U}{\sqrt{2}}\sqrt{|\mathcal{Z}|^2 + |\tilde{\mathcal{Z}}|^2 + e^{-2U}{r_0}^2 + \sqrt{\left(|\mathcal{Z}|^2 + |\tilde{\mathcal{Z}}|^2 + e^{-2U}{r_0}^2\right)^2 - 4|\mathcal{Z}|^2|\tilde{\mathcal{Z}}|^2}}\,,\\ \Psi_i &= e^{2U}\frac{\mathcal{Z}^*\,\mathcal{D}_i\mathcal{Z}}{\Upsilon}\,,\end{aligned}$$

In this case we can write

$$-\left[e^{2U} V_{\rm bh} - {r_0}^2\right] = \Upsilon^2 + 4\,\mathcal{G}^{ij^*}\,\Psi_i \Psi^*_{j^*}\,,$$

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$$\Psi_i = e^{2U}\,\frac{\mathcal{Z}^*\,\mathcal{D}_i\mathcal{Z}}{\Upsilon}\,,$$

Since

$$\partial_U\,\Psi_i - \partial_i\Upsilon = \partial_i\Psi_j - \partial_j\Psi_i = \partial_{i*}\Psi_j - \partial_j\Psi^*_{i^*} = 0\,,$$

there exists a generalized superpotential, whose gradient generates the vector field $(\Upsilon, \Psi_i, \Psi^*_{j^*})$ and the first-order equations

$$U' = \Upsilon\,, \qquad Z^{i\,\prime} = 2\,\mathcal{G}^{ij^*}\,\Psi^*_{j^*}\,.$$

although it is very difficult to find explicitly.

The extremal case

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We start by calculating the critical points of the black-hole potential:

$$\mathcal{G}^{ij^*}\partial_{j^*}V_{\rm bh} = 2\,Z^{\Lambda}\Gamma_{\Lambda}\left(\Gamma^{*\,i} - \Gamma^{*\,0}Z^{i}\right) = 0 \quad \Rightarrow \left\{ \begin{array}{l} Z^{i}{}_{\rm h} = \Gamma^{*\,i}/\Gamma^{*\,0}\,, \\ (\text{isolated, supersymmetric attractor}) \\ \\ Z^{\Lambda}{}_{\rm h}\Gamma_{\Lambda} = 0\,, \\ (\text{non} - \text{supersymmetric hypersurface}) \end{array} \right.$$

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Attractor	$e^{-\mathcal{K}_{\rm h}}$	$\lvert\mathcal{Z}_{\rm h}\rvert^2$	$\lvert\tilde{\mathcal{Z}}_{\rm h}\rvert^2$	$-V_{\rm bhh}$	M
$Z^{i\,\rm susy}_{\rm h} = \Gamma^{*\,i}/\Gamma^{*\,0}$	$\Gamma^{*\,\Lambda}\Gamma_\Lambda > 0$	$\Gamma^{*\,\Lambda}\Gamma_\Lambda$	0	$\Gamma^{*\,\Lambda}\Gamma_\Lambda$	$\lvert\mathcal{Z}_\infty\rvert$
$Z^{\Lambda\,\rm nsusy}_{\rm h}\Gamma_\Lambda = 0$	$-\Gamma^{*\,\Lambda}\Gamma_\Lambda > 0$	0	$-\Gamma^{*\,\Lambda}\Gamma_\Lambda$	$-\Gamma^{*\,\Lambda}\Gamma_\Lambda$	$\lvert\tilde{\mathcal{Z}}_\infty\rvert$

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Defining the complex combinations of harmonic functions

$$\mathcal{H}_{\Lambda} \equiv \mathcal{I}_{\Lambda} + \tfrac{i}{2}\eta_{\Lambda\Sigma}\mathcal{I}^{\Sigma} \equiv \mathcal{H}_{\Lambda\,\infty} - \tfrac{1}{\sqrt{2}}\Gamma_{\Lambda}\tau\,,$$

we find the form of the metric and the complex scalar fields in terms of those harmonic functions

$$e^{-2U} = 2\mathcal{H}^{*\,\Lambda}\mathcal{H}_{\Lambda}\,, \qquad Z^{i} = \frac{\mathcal{R}^{i} + i\mathcal{I}^{i}}{\mathcal{R}^{0} + i\mathcal{I}^{0}} = \frac{\mathcal{H}^{*i}}{\mathcal{H}^{*0}}\,.$$

The solution depends on the $n+1$ charges Γ_Λ and on the $n+1$ constants $\mathcal{H}_{\Lambda\,\infty}$. these are determined from

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The complete supersymmetric solution is, therefore, given by the $n+1$ complex harmonic functions

$$\mathcal{H}^{\mathrm{susy}}{}_\Lambda = e^{\mathcal{K}_\infty/2} \frac{\mathcal{Z}_\infty}{|\mathcal{Z}_\infty|} Z^*_{\Lambda\,\infty} - \tfrac{1}{\sqrt{2}} \Gamma_\Lambda \tau \,,$$

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Our ansatz for the non-extremal solution is

$$e^{-2U} = e^{-2[U_{\rm e}(\hat{\mathcal{H}})+r_0\tau]}\,, \qquad e^{-2U_{\rm e}(\hat{\mathcal{H}})} = 2\hat{\mathcal{H}}^{*\,\Lambda}\hat{\mathcal{H}}_{\Lambda}\,, \qquad Z^i = Z^i{}_{\rm e}(\hat{\mathcal{H}}) = \hat{\mathcal{H}}^{*\,i}/\hat{\mathcal{H}}^{*\,0}\,,$$

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The $2(n+1)$ complex constants A_{Λ}, B_{Λ} are found by requiring our Ansatz to solve the e.o.m. ($f \equiv e^{r_0\tau}$)

$$\begin{aligned}
\ddot{U}_{\rm e} - (\dot{U}_{\rm e})^2 - \mathcal{G}_{ij^*}\dot{Z}^i\dot{Z}^{*\,j^*} &= 0\,,\\
(2r_0)^2\left[f\ddot{U}_{\rm e} + \dot{U}_{\rm e}\right] + e^{2U_{\rm e}}V_{\rm bh} &= 0\,,\\
(2r_0)^2\left[f\left(\ddot{Z}^i + \mathcal{G}^{ij^*}\partial_k\mathcal{G}_{lj^*}\dot{Z}^k\dot{Z}^l\right) + \dot{Z}^i\right] + e^{2U_{\rm e}}\mathcal{G}^{ij^*}\partial_{j^*}V_{\rm bh} &= 0\,.
\end{aligned}$$

The e.o.m. are solved if the the constants satisfy the algebraic equations

$$\begin{aligned}
\Im\mathrm{m}(B^{*\,\Lambda}A_\Lambda) &= 0\,,\\
A^{*\,\Lambda}A^\Sigma\xi_{\Lambda\Sigma} &= 0\,,\\
(A^{*\,\Lambda}B^\Sigma + B^{*\,\Lambda}A^\Sigma)\xi_{\Lambda\Sigma} &= 0\,,\\
B^{*\,\Lambda}B^\Sigma\xi_{\Lambda\Sigma} &= 0\,,\\
(2r_0)^2(B^*_iA^*_0 - B^*_0A^*_i)A^{*\,\Lambda}A_\Lambda + (\Gamma^*_iA^*_0 - \Gamma^*_0A^*_i)A^{*\,\Lambda}\Gamma_\Lambda &= 0\,,\\
-(2r_0)^2(B^*_iA^*_0 - B^*_0A^*_i)B^{*\,\Lambda}B_\Lambda + (\Gamma^*_iB^*_0 - \Gamma^*_0B^*_i)B^{*\,\Lambda}\Gamma_\Lambda &= 0\,,\\
(\Gamma^*_iA^*_0 - \Gamma^*_0A^*_i)A^{*\,\Lambda}\Gamma_\Lambda + (\Gamma^*_iB^*_0 - \Gamma^*_0B^*_i)B^{*\,\Lambda}\Gamma_\Lambda &= 0\,,
\end{aligned}$$

where we have defined

$$\xi_{\Lambda\Sigma} \equiv 2\left(\Gamma_\Lambda\Gamma^*_\Sigma + 8{r_0}^2A_\Lambda B^*_\Sigma\right) - \eta_{\Lambda\Sigma}\left(\Gamma^\Omega\Gamma^*_\Omega + 8{r_0}^2A^\Omega B^*_\Omega\right)\,.$$

Furthermore, we need to normalize the metric at spatial infinity and relate A_Λ, B_Λ to the physical parameters:

$$
\begin{aligned}
2(A^{*\,\Lambda} + B^{*\,\Lambda})(A_\Lambda + B_\Lambda) &= 1\,, \\
4\Re\mathrm{e}[B^{*\,\Lambda}(A_\Lambda + B_\Lambda)] &= 1 - M/r_0\,, \\
\frac{A^{*\,i} + B^{*\,i}}{A^{*\,0} + B^{*\,0}} &= Z^i{}_\infty\,.
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Up to a phase to be determined in the supersymmetric extremal limit the solution is

$$\begin{aligned} A_\Lambda &= \pm\frac{e^{\mathcal{K}_\infty/2}}{2\sqrt{2}}\left\{Z^*_{\Lambda\,\infty}\left[1+\frac{(M^2-e^{\mathcal{K}_\infty}|Z^{*\,\Sigma}_\infty\Gamma^*_\Sigma|^2)}{Mr_0}\right]+\frac{\Gamma_\Lambda Z^{*\,\Sigma}{}_\infty\Gamma^*_\Sigma}{Mr_0}\right\}, \\ B_\Lambda &= \pm\frac{e^{\mathcal{K}_\infty/2}}{2\sqrt{2}}\left\{Z^*_{\Lambda\,\infty}\left[1-\frac{(M^2-e^{\mathcal{K}_\infty}|Z^{*\,\Sigma}_\infty\Gamma^*_\Sigma|^2)}{Mr_0}\right]-\frac{\Gamma_\Lambda Z^{*\,\Sigma}_\infty\Gamma^*_\Sigma}{Mr_0}\right\}, \\ M^2{r_0}^2 &= (M^2-|\mathcal{Z}_\infty|^2)(M^2-|\tilde{\mathcal{Z}}_\infty|^2)\,. \end{aligned}$$

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The metric is regular in all the $r_0{}^2 > 0$ cases.

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1. Supersymmetric , when $M^2 \to |\mathcal{Z}|^2 = e^{\mathcal{K}_\infty} |Z^\Sigma_\infty \Gamma_\Sigma|^2$. We get

$$\hat{\mathcal{H}}_\Lambda \stackrel{M \to |\mathcal{Z}_\infty|}{\longrightarrow} \pm \frac{\mathcal{Z}^*_\infty}{|\mathcal{Z}_\infty|} \mathcal{H}^{\rm susy}{}_\Lambda \,,$$

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On the event horizon the scalars take the values

$$Z^{*\,i}_{\text{h}} = \frac{\Gamma^i Z^{*\,\Lambda}_\infty \Gamma^*_\Lambda - Z^{*\,i}_\infty \Gamma^{*\,\Sigma}\Gamma_\Sigma}{\Gamma^0 Z^{*\,\Gamma}_\infty \Gamma^*_\Gamma - \Gamma^{*\,\Omega}\Gamma_\Omega} \,,$$

which depend manifestly on the asymptotic values (so there is no attractor behavior in this case).

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One can compute the "entropies" of the inner and outer horizons (event horizon (+) and Cauchy horizon):

$$\frac{S_\pm}{\pi} = (M^2 - |\mathcal{Z}_\infty|^2) \pm (M^2 - |\tilde{\mathcal{Z}}_\infty|^2) \pm 2Mr_0\,.$$

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They can also be written in the suggestive form

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The product of these "entropies" S_+S_- is manifestly moduli-independent for all values of r_0.

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We can speak of an attractor behavior in the evaporation process.

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- ★ We have seen how the supersymmetric solutions can be deformed into non-extremal ones from which one can recover different extremal solutions (supersymmetric and non-supersymmetric) with an explicit example.

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