Supersymmetric non-Abelian monopoles and black holes in N=2,d=4 Super-Einstein-Yang-Mills Theories

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Based on 0802.1799 and 0806.1477.

Work done in collaboration with *M. Hübscher, P. Meessen and S. Vaulà* (IFT UAM/CSIC, Madrid)

Plan of the Talk:

1 – Introduction

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Now it is natural to ask what happens in the gauged theories. There are several possible gaugings in $N = 2, d = 4$ theories. Let's review the theory.

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We are not going to consider hypermultiplets in this seminar.

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The spinors are *sections* of the bundle: under Kähler transformations

$$\delta_f\mathcal{K} = f(Z) + f^*(Z^*)\,, \qquad \delta_f\psi_{I\mu} = -\tfrac{1}{4}[f(Z) - f^*(Z^*)]\psi_{I\mu}\,,$$

and their covariant derivatives contain the pullback of the Kähler connection 1-form $\hat{\mathcal{Q}} \equiv \mathcal{Q}_i dZ^i + \mathcal{Q}_{i^*} dZ^{*i^*}$

$$\mathcal{D}_\mu\psi_{I\nu} = \{\nabla_\mu + \tfrac{i}{2}\mathcal{Q}_\mu\}\psi_{I\nu}\,.$$

Local $N = 2$ supersymmetry requires the Kähler-Hodge manifold to be a special Kähler manifold, so it is the base space of a $2(n_V + 1)$-dimensional vector bundle with $Sp[2(n_V + 1), \mathbb{R}]$ structure group, on which we can define the constrained symplectic section

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These three elements are not independent. They are related by the constraints of special Kähler geometry. They can also be derived from a prepotential.

The action of the bosonic fields

The action of the bosonic fields of the ungauged theory is

$$S = \int d^4x \sqrt{|g|} \left[R + 2\mathcal{G}_{ij^*}\partial_\mu Z^i \partial^\mu Z^{*\,j^*} + 2\Im\mathrm{m}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda\,\mu\nu} F^{\Sigma}{}_{\mu\nu} - 2\Re\mathrm{e}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda\,\mu\nu} \star F^{\Sigma}{}_{\mu\nu} \right] .$$

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These theories have supersymmetric, extreme, black holes, with Abelian charges. In order to study the situation with non- Abelian vector fields we must gauge these theories.

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We are going to see that, if we do not add hypermultiplets there are just three possibilities:

1. We gauge an $U(1)$ subgroup of the $SU(2) \subset SU(2) \times U(1)$ R-symmetry group, using Fayet-Iliopoulos terms.
2. We gauge a subgroup G of the isometry group of the special Kähler manifold in combination with **the** $U(1)$ subgroup of the R-symmetry group.
3. If G contains an $SU(2)$ factor we can combine this gauging with the $SU(2)$ subgroup of the R-symmetry group by using $SU(2)$ Fayet-Iliopoulos terms.

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- global isometries of the special Kähler metric $\mathcal{G}_{ij^*}$.

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This makes it possible to always gauge the R-symmetry $U(1) \subset SU(2)$ using just one vector field (*Fayet-Iliopoulos* terms). In order to gauge the full $SU(2)$ the vector multiplets must have this symmetry (see below).

Additionally, it *may* have the following invariances:

- global $SO(n_V+1)$ rotations of the vectors ($Sp[2(n_V+1),\mathbb{R}]$ in the e.o.m.).
- global isometries of the special Kähler metric $\mathcal{G}_{ij^*}$.

These transformations are not independent due to $\mathcal{N}_{\Lambda\Sigma}$.

The global symmetries to be gauged

The ungauged theory has, by construction, the following symmetries (apart from g.c.t.s)

- global R-symmetry $SU(2)\times U(1)$: $\left\{\begin{array}{lcl} U(1) & \longrightarrow & \psi'_{I\mu} = e^{\frac{i}{4}\beta}\psi_{I\mu}\,, \\ SU(2) & \longrightarrow & \psi'_{I\mu} = \Lambda_I{}^J\psi_{J\mu}\,. \end{array}\right.$
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These transformations are not independent due to $\mathcal{N}_{\Lambda\Sigma}$. Furthermore, ordinary isometries are not symmetries of the full theory:

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$$\delta_{\alpha} Z^{i} = \alpha^{\Lambda} k_{\Lambda}{}^{i}(Z)\,, \qquad [K_{\Lambda}, K_{\Sigma}] = -f_{\Lambda\Sigma}{}^{\Omega} K_{\Omega}\,,$$

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$$\delta_\alpha A^\Lambda{}_\mu = \alpha^\Sigma f_{\Sigma\Omega}{}^\Lambda A^\Omega{}_\mu\,, \qquad \delta_\alpha \mathcal{N}_{\Lambda\Sigma} = -2\alpha^\Omega f_{\Omega(\Lambda}{}^\Gamma \mathcal{N}_{\Sigma)\Gamma}\,.$$

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→ The Kähler structure will be preserved if

1. The Kähler potential is preserved (up to Kähler transformations)

$$\pounds_\Lambda \mathcal{K} \equiv k_\Lambda{}^i \partial_i \mathcal{K} + k^*_\Lambda{}^{i^*} \partial_{i^*} \mathcal{K} = \lambda_\Lambda(Z) + \lambda^*_\Lambda(Z^*)\,.$$

2. The Kähler 2-form $\mathcal{J} = i\mathcal{G}_{ij^*} dZ^i \wedge dZ^{*j^*}$ is also preserved:

$$\pounds_\Lambda \mathcal{J} = 0\,.$$

Then,

$$\left.\begin{array}{l} d\mathcal{J} = 0 \;\; \Rightarrow \pounds_{\Lambda}\mathcal{J} = d(i_{k_\Lambda}\mathcal{J})\,, \\[2em] \pounds_{\Lambda}\mathcal{J} = 0\,, \end{array}\right\} \Rightarrow d(i_{k_\Lambda}\mathcal{J}) = 0\,, \;\; \Rightarrow i_{k_\Lambda}\mathcal{J} = d\mathcal{P}_{\Lambda}\,, \Leftrightarrow k_{\Lambda\, i^*} = i\partial_{i^*}\mathcal{P}_{\Lambda}\,.$$

for some real 0-forms $\mathcal{P}_\Lambda$: the *momentum maps* or *Killing prepotentials*.

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→ The preservation of the Hodge structure requires that we accompany the transformations δ_α with $U(1)$ transformations. In particular, the spinors must transform as

$$\delta_\alpha \psi_{I\mu} = -\tfrac{1}{4}\alpha^{\Lambda}(\lambda_\Lambda - \lambda^*_\Lambda)\psi_{I\mu}\,,$$

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→ The preservation of the special Kähler structure requires that the symplectic section transforms as

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Summarizing, we can gauge

1. (Always) A $U(1)$ subgroup of the R-symmetry group, via Fayet-Iliopoulos terms. The timelike supersymmetric solutions of these theories have been classified in Caldarelli & Klemm, `hep-th/0307022`, Cacciatori, Caldarelli, Klemm & Mansi, `hep-th/0406238`, Cacciatori, Caldarelli, Klemm, Mansi & Roest, `arXiv:0704.0247` [hep-th] and Cacciatori, Klemm, Mansi & Zorzan, `arXiv:0804.0009 [hep-th]`.

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3 − $N = 2, d = 4$ SEYM

To gauge the theory we replace the standard by gauge-covariant derivatives

$$\partial_\mu Z^i \quad \longrightarrow \quad \mathfrak{D}_\mu Z^i \equiv \partial_\mu Z^i + g A^\Lambda{}_\mu k_\Lambda{}^i \,,$$

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The action of the bosonic fields takes the form

$$\begin{aligned} S \quad = \quad & \int d^4x \sqrt{|g|}\, \big[R + 2\mathcal{G}_{ij^*}\mathfrak{D}_\mu Z^i \mathfrak{D}^\mu Z^{*\,j^*} + 2\Im\mathrm{m}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda\,\mu\nu} F^\Sigma{}_{\mu\nu} \\ & -2\Re\mathrm{e}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda\,\mu\nu\star} F^\Sigma{}_{\mu\nu} - V(Z, Z^*)\big]\,, \end{aligned}$$

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where the potential is given by

$$V(Z,Z^*) = -\tfrac{1}{4}g^2\Im\mathrm{m}\mathcal{N}^{-1|\Lambda\Sigma}\mathcal{P}_\Lambda\mathcal{P}_\Sigma \;\geq\; 0\,.$$

(just as in $N = 1$ without superpotential!)

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\end{aligned}$$

Our goal is to find, for all possible $N = 2, d = 4$ SEYM theories all the bosonic field configurations $e^a{}_\mu(x), A^\Lambda{}_\mu(x), Z^i(x)$ that admit Killing spinors and then impose the equations of motion to find supersymmetric solutions.

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3. Finding consistency/integrability conditions from these equations to determine the general form of the supersymmetric configurations.
4. Proving directly that all those configurations that satisfy the necessary conditions indeed admit Killing spinors.

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2. Constructing bilinears (complex scalar $X = \frac{1}{2}\varepsilon_{IJ}\bar{\epsilon}^I\epsilon^J$, 4 real vectors, $V^I{}_J{}^\mu = i\bar{\epsilon}^I\gamma^\mu\epsilon_J$, and 3 anti-self-dual 2-forms $\Phi_{IJ\,\mu\nu} = \bar{\epsilon}_I\gamma_{\mu\nu}\epsilon_J$) and find which equations they must satisfy if ϵ is a Killing spinor.
3. Finding consistency/integrability conditions from these equations to determine the general form of the supersymmetric configurations.
4. Proving directly that all those configurations that satisfy the necessary conditions indeed admit Killing spinors.
5. Imposing the independent equations of motion to find all the supersymmetric solutions.

Our goal is to find, for all possible $N = 2, d = 4$ SEYM theories all the bosonic field configurations $e^a{}_\mu(x), A^\Lambda{}_\mu(x), Z^i(x)$ that admit Killing spinors and then impose the equations of motion to find supersymmetric solutions.

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This method does not *classify* the supersymmetric configurations by their number of independent Killing spinors. It should be supplemented by the spinorial geometry method of Papadopoulos, Gran, Roest, Gutowski et al.

General results

In general, the vector bilinear $V^{\mu} \equiv V^{I}{}_{I}{}^{\mu}$ is a Killing vector (consistency condition) that can be timelike or null, providing a preliminary classification of the configurations. In general

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The timelike class contains very interesting non-Abelian generalizations of the Abelian black-hole solutions.

We are going to focus on this case.

Our results for the timelike case can be summarized in the following

RECIPE:

☞ Find a set of Yang-Mills fields $\tilde{A}^{\Lambda}{}_{m}$ and functions $\mathcal{I}^{\Lambda}$ in $\mathbb{R}^3$ satisfying

$$\tfrac{1}{2}\,\epsilon_{xyz}\,\tilde{F}^{\Lambda}{}_{\underline{xy}} \;=\; -\frac{1}{\sqrt{2}}\,\tilde{\mathfrak{D}}_{\underline{z}}\mathcal{I}^{\Lambda}\;,$$

which is the Bogomol'nyi equation satisfied by known magnetic monopole solutions.

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☞ Use the above solution to find a solution of

$$\tilde{\mathfrak{D}}_{m}\tilde{\mathfrak{D}}_{m}\mathcal{I}_{\Lambda} = \frac{1}{2}g^{2}\,\left[f_{\Lambda(\Sigma}{}^{\Gamma}f_{\Delta)\Gamma}{}^{\Omega}\,\mathcal{I}^{\Sigma}\mathcal{I}^{\Delta}\right]\,\mathcal{I}_{\Omega}\,,$$

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$$\mathcal{I}_{\Lambda} \propto \mathcal{I}^{\Lambda}\,.$$

The real symplectic vector $\mathcal{I} = (\mathcal{I}^{\Lambda}, \mathcal{I}_{\Lambda})$ determines completely the solution. The physical fields $g_{\mu\nu}, A^{\Lambda}{}_{\mu}, Z^{i}$ are derived from them as follows:

☞ Solve the stabilization equations to find $\mathcal{R}^{\Lambda}$ and $\mathcal{R}_{\Lambda}$. N.B.:

$$
\begin{aligned}
\mathcal{I}^{\Lambda} &\equiv \Im\mathrm{m}(\mathcal{L}^{\Lambda}/X)\,, & \mathcal{I}_{\Lambda} &\equiv \Im\mathrm{m}(\mathcal{M}_{\Lambda}/X)\,,\\
\mathcal{R}^{\Lambda} &\equiv \Re\mathrm{e}(\mathcal{L}^{\Lambda}/X)\,, & \mathcal{R}_{\Lambda} &\equiv \Re\mathrm{e}(\mathcal{M}_{\Lambda}/X)\,.
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☞ and compute

$$2|X|^{2} = \langle\, \mathcal{R} \mid \mathcal{I}\,\rangle^{-1}\,.$$

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$$A^{\Lambda}{}_{\mu} dx^{\mu} = -\sqrt{2}|X|^2 \mathcal{R}^{\Lambda}(dt + \hat{\omega}) + \tilde{A}^{\Lambda}{}_{\underline{x}} dx^{x} \ ,$$

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$SO(3)$ Examples:

Let us consider $N = 2$ EYM systems containing an $SO(3)$ gauge group, with indices $a = 1, 2, 3$.

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$$\mathcal{I}^a = \mathcal{I}\, n^a\,, \qquad A^a{}_m = \Phi\, \varepsilon_{mb}{}^a\, n^b\,, \qquad n^a \equiv x^a/r\,, \quad r \equiv \sqrt{x^b x^b}\,.$$

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A 2-parameter (μ and ρ) family of solutions is given by

$$\mathcal{I}(r) = \frac{\sqrt{2}\mu}{g}\mathsf{H}_\rho(\mu r)\,, \quad \mathsf{H}_\rho(r) = \coth(r+\rho) - \frac{1}{r}\,,$$

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The two most interesting cases are $\rho = 0, \infty$.

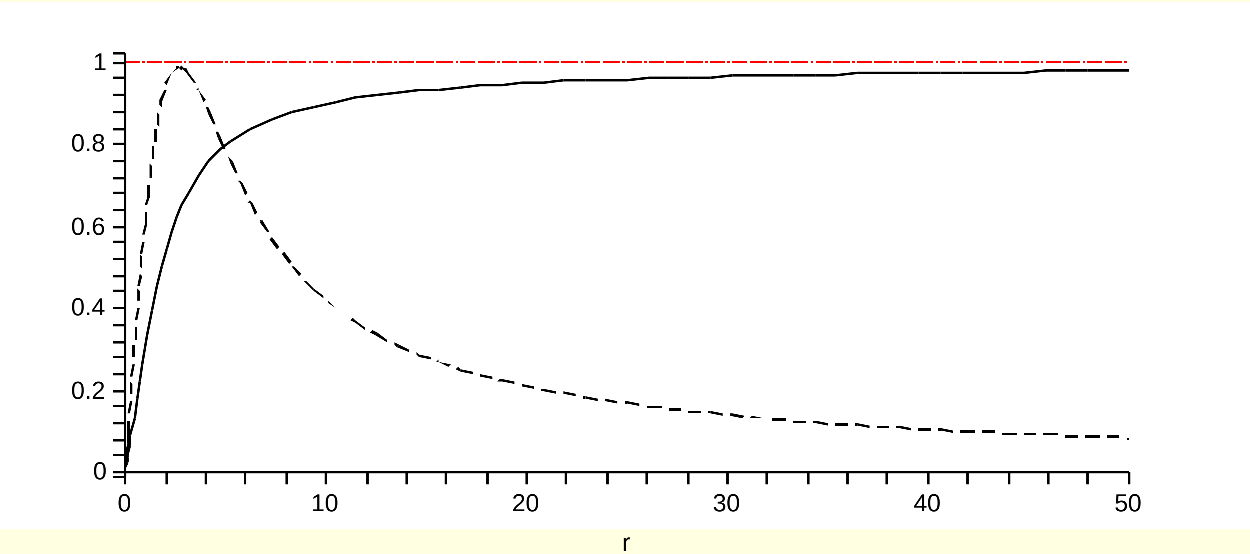
6 – 't Hooft-Polyakov Monopoles

The $\rho = 0$ solution can be written in the form

$$A^a{}_m = \varepsilon_{mb}{}^a\, n^b\, \frac{\mu}{g}\, \mathsf{G}_0(\mu r)\,, \quad \mathsf{G}_0(r) = \frac{1}{r} - \frac{1}{\sinh r}\,,$$

$$\mathcal{I}^a = \frac{\sqrt{2}\mu}{g}\, \mathsf{H}_0(\mu r)\, n^a\,, \qquad \mathsf{H}_0(r) = \coth r - \frac{1}{r}\,.$$

The profiles of the functions G and H are



$\mathcal{I}^a$ is regular at $r = 0$ for $\rho = 0$, and describes the 't Hooft-Polyakov monopole.

7 – Black Hedgehogs

In the limit $\rho \to \infty$ we find the "black hedgehog" solution

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\begin{aligned}
\mathcal{I}^a &= -\sqrt{2}\left(\mathcal{I}_\infty + \frac{1}{gr}\right) n^a \,, \\
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The possible existence of an event horizon covering the singularity at $r = 0$ has to be studied in specific models.

Before finding $\mathcal{R}$ and $|X|$ we have to find the $\mathcal{I}_a$s solving

$$\mathfrak{D}_m \mathfrak{D}_m \mathcal{I}_\Lambda \;=\; \tfrac{1}{2} g^2 \;\left[f_{\Lambda(\Sigma}{}^{\Gamma} f_{\Delta)\Gamma}{}^{\Omega}\, \mathcal{I}^{\Sigma} \mathcal{I}^{\Delta} \right]\, \mathcal{I}_\Omega \;,$$

and solve the staticity constraint

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where $\mathcal{J}$ is an arbitrary constant.

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where $\mathcal{J}$ is an arbitrary constant.

If we split the index Λ into an a-index and an u-index labeling the ungauged directions, the staticity constraint only acts non-trivially on the ungauged part:

$$\mathcal{I}_u \, d\mathcal{I}^u - \mathcal{I}^u \, d\mathcal{I}_u + \mathcal{I}_a \, \mathfrak{D}\mathcal{I}^a - \mathcal{I}^a \, \mathfrak{D}\mathcal{I}_a = \mathcal{I}_u \, d\mathcal{I}^u - \mathcal{I}^u \, d\mathcal{I}_u = 0 ,$$

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which we can solve as in the Abelian case or just set to zero.

This determines completely the family of solutions but, in order to find explicit expressions for $\mathcal{R}$ and $|X|$ and the spacetime metric we must solve the stabilization equations which depend on the specific model considered.

Metrics

For simplicity let us consider a $\overline{\mathbb{CP}}^3$ model whose prepotential reads

$$\mathcal{F} \;=\; \tfrac{i}{4}\,\eta_{\Lambda\Sigma}\,\mathcal{X}^{\Lambda}\,\mathcal{X}^{\Sigma}\,, \qquad \eta \;=\; \mathrm{diag}\,(\,-\,,\;[+]^{n}\,)\;.$$

The Kähler potential is

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With the hedgehog Ansatz $\mathcal{I}^{a\,2} = \mathcal{I}^{2}$ and $SU(2)$ effectively reduces to a $U(1)$ in the metric!

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With the hedgehog Ansatz $\mathcal{I}^{a\,2} = \mathcal{I}^2$ and $SU(2)$ effectively reduces to a $U(1)$ in the metric! For black holes with finite entropy (attractor) we need at least two $U(1)$s. However, since $\mathcal{I}^a$ is bound in the monopole, we do not need $\mathcal{I}^0, \mathcal{I}_0$ and we can set them to constants.

Normalizing to have asymptotic flatness, we get, for the monopole

$$-g_{rr} = 1 \; + \; \mu^2 \left[\frac{1}{g^2} \; + \; \mathcal{J}^2\right] \; \left[1 \; - \; \mathsf{H}^2(\mu r)\right] \; ,$$

which is completely regular and describes an object of mass

$$\mathsf{M} \; = \; \mu \left[1/g^2 \; + \; \mathcal{J}^2\right] \; .$$

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To embed the black hedgehog into this model and get a regular solution ($|Z|^2 < 1$) we need non-trivial $\mathcal{I}^0$ or $\mathcal{I}_0$. The conditions for regularity are the same as in an standard, Abelian $U(1) \times U(1)$ black hole of this model:

$$\mathsf{M} \;=\; \mathcal{I}^0_\infty p^0 + \mathcal{I}_{0\infty} q_0 - 2\mu \left[1/g^2 \;+\; \mathcal{J}^2\right] > 0 \,,$$

$$\frac{A}{4\pi} \;=\; \tfrac{1}{2}\left[(p^0)^2 + 4(q_0)^2\right] - 2\frac{\mu^2}{g^2} \left[1/g^2 \;+\; \mathcal{J}^2\right] > 0 \,,$$

and can always be satisfied.

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(related to Harvey & Liu (1991) and Chamseddine & Volkov (1997) monopole solutions.)

To embed the black hedgehog into this model and get a regular solution ($|Z|^2 < 1$) we need non-trivial $\mathcal{I}^0$ or $\mathcal{I}_0$. The conditions for regularity are the same as in an standard, Abelian $U(1) \times U(1)$ black hole of this model:

$$\mathsf{M} \;\; = \;\; \mathcal{I}^0_\infty p^0 + \mathcal{I}_{0\infty} q_0 - 2\mu \left[1/g^2 \; + \; \mathcal{J}^2\right] > 0 \, ,$$

$$\frac{A}{4\pi} \;\; = \;\; \tfrac{1}{2}[(p^0)^2 + 4(q_0)^2] - 2\frac{\mu^2}{g^2} \left[1/g^2 \; + \; \mathcal{J}^2\right] > 0 \, ,$$

and can always be satisfied.

How does the attractor mechanism work in this solution?

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- There is still much work to do to classify all the possible supersymmetric solutions....

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THANKS!