Supersymmetric solutions of N=1 and N=2 d=4 supergravities

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Based on `arXiv:0712.1530, 0802.1799 and work in progress.`

Work done in collaboration with *M. Hübscher, P. Meessen and S. Vaulà* (IFT UAM/CSIC, Madrid)

Plan of the Talk:

1 I All the supersymmetric solutions of $N = 1, d = 4$ SUGRA

2 Review of $N = 1, d = 4$ SUGRA: 1.- the ungauged theory

5 Review of $N = 1, d = 4$ SUGRA: 2.- the gauged theory

9 The supersymmetric solutions of $N = 1, d = 4$ SUGRAs

10 The supersymmetric solutions of $N = 1, d = 4$ SUGRAs

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$1 - $ I All the supersymmetric solutions of $N = 1, d = 4$ SUGRA

Most of the work done on the classification of supersymmetric solutions of SUGRA has been done in $d > 4$ and $N > 1$. However, $N = 1, d = 4$ SUGRA is a theory with more direct phenomenologial interest and some particular examples of supersymmetric solutions are known.

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Thus, we start by reviewing the most general $N = 1, d = 4$ SUGRA theory.

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All fermions are represented by chiral 4-component spinors: $\gamma_5 \psi_{\mu} = -\psi_{\mu}$ etc.

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The spinors are *sections* of the bundle: under Kähler transformations

$$\delta_f\mathcal{K} = f(Z) + f^*(Z^*) \,, \qquad \delta_f\lambda^\Lambda = -\tfrac{1}{4}(f(Z) - f^*(Z^*))\lambda^\Lambda \,,$$

and their covariant derivatives contain the pullback of the Kähler connection 1-form $\hat{\mathcal{Q}} \equiv \mathcal{Q}_i dZ^i + \mathcal{Q}_{i^*}dZ^{*i^*}$

$$\mathcal{D}_\mu\lambda^\Lambda = \{\partial_\mu - \tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} + \tfrac{i}{2}\mathcal{Q}_\mu\}\lambda^\Lambda \,.$$

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The action of the bosonic fields is then given by

$$S = \int d^4x \sqrt{|g|} \left[R + 2\mathcal{G}_{ij^*} \partial_\mu Z^i \partial^\mu Z^{*\, j^*} - \Im\mathrm{m} f_{\Lambda\Sigma} F^{\Lambda\, \mu\nu} F^{\Sigma}{}_{\mu\nu} - \Re\mathrm{e} f_{\Lambda\Sigma} F^{\Lambda\, \mu\nu} \star F^{\Sigma}{}_{\mu\nu} - V(Z, Z^*) \right] .$$

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The scalar potential $V(Z,Z^*)$ is determined by the superpotential and Kähler metric:

$$V(Z,Z^*)=-24|\mathcal{L}|^2+8\mathcal{G}^{ij^*}\mathcal{D}_i\mathcal{L}\mathcal{D}_{j^*}\mathcal{L}^*\,,$$

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→ The Kähler structure will be preserved if

1. The Kähler potential is preserved (up to Kähler transformations)

$$\pounds_{\Lambda}\mathcal{K} \equiv k_{\Lambda}{}^{i}\partial_i\mathcal{K} + k^{*}_{\Lambda}{}^{i^*}\partial_{i^*}\mathcal{K} = \lambda_{\Lambda}(Z) + \lambda^{*}_{\Lambda}(Z^*)\,.$$

2. The Kähler 2-form $\mathcal{J} = i\mathcal{G}_{ij^*} dZ^i \wedge dZ^{*j^*}$ is also preserved:

$$\pounds_{\Lambda}\mathcal{J} = 0\,.$$

Then,

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for some real 0-forms $\mathcal{P}_\Lambda$ known as *momentum maps* or *Killing prepotentials*.

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→ The preservation of the Hodge structure requires that we accompany the transformations δ_α with $U(1)$ transformations. In particular, the superpotential $\mathcal{L}$ must transform under the isometries as

$$\delta_\alpha \mathcal{L} = -\tfrac{1}{2}\alpha^\Lambda(\lambda_\Lambda - \lambda^*_\Lambda)\mathcal{L}\,,$$

and, analogously, the spinors must transform as

$$\delta_\alpha \psi_\mu = -\tfrac{1}{4}\alpha^\Lambda(\lambda_\Lambda - \lambda^*_\Lambda)\psi_\mu\,,$$

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$$\left.\begin{array}{l} d\mathcal{J} = 0 \ \ \Rightarrow \pounds_\Lambda \mathcal{J} = d(i_{k_\Lambda}\mathcal{J})\,, \\ \\ \pounds_\Lambda \mathcal{J} = 0\,, \end{array}\right\} \Rightarrow d(i_{k_\Lambda}\mathcal{J}) = 0\,, \ \Rightarrow i_{k_\Lambda}\mathcal{J} = d\mathcal{P}_\Lambda\,, \Leftrightarrow k_{\Lambda\, i^*} = i\partial_{i^*}\mathcal{P}_\Lambda\,.$$

for some real 0-forms $\mathcal{P}_\Lambda$ known as *momentum maps* or *Killing prepotentials*. The momentum maps are defined up to an additive real constant. In $N = 1, d = 4$ theories (but **not** in $N = 2, d = 4$) it is possible to have non-vanishing, constant, momentum maps for vanishing Killing vectors, giving rise to *Fayet-Iliopoulos* terms.

→ The preservation of the Hodge structure requires that we accompany the transformations δ_α with $U(1)$ transformations. In particular, the superpotential $\mathcal{L}$ must transform under the isometries as

$$\delta_\alpha \mathcal{L} = -\tfrac{1}{2}\alpha^\Lambda(\lambda_\Lambda - \lambda^*_\Lambda)\mathcal{L}\,,$$

and, analogously, the spinors must transform as

$$\delta_\alpha \psi_\mu = -\tfrac{1}{4}\alpha^\Lambda(\lambda_\Lambda - \lambda^*_\Lambda)\psi_\mu\,,$$

→ If these conditions are met, this is a global symmetry of the theory that we can gauge.

To gauge the theory we replace the standard derivatives by gauge-covariant derivatives

$$\partial_\mu Z^i \quad \longrightarrow \quad \mathfrak{D}_\mu Z^i \equiv \partial_\mu Z^i + g A^\Lambda{}_\mu k_\Lambda{}^i\,,$$

$$\mathcal{D}_\mu \psi_\nu \quad \longrightarrow \quad \mathfrak{D}_\mu \psi_\nu \equiv \{\nabla_\mu + \tfrac{i}{2}\hat{\mathcal{Q}}_\mu\}\psi_\nu$$

where

$$\hat{\mathcal{Q}}_\mu \equiv \mathcal{Q}_\mu + g A^\Lambda{}_\mu \mathcal{P}_\Lambda \neq \mathcal{Q}_i\, \mathfrak{D}_\mu Z^i + \mathcal{Q}_{i^*}\, \mathfrak{D}_\mu Z^{i^*}\,.$$

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The action of the bosonic fields takes the form

$$S = \int d^4x \sqrt{|g|} \left[R + 2\mathcal{G}_{ij^*} \mathfrak{D}_\mu Z^i \mathfrak{D}^\mu Z^{*\,j^*} - \Im\mathrm{m} f_{\Lambda\Sigma} F^{\Lambda\,\mu\nu} F^\Sigma{}_{\mu\nu} - \Re\mathrm{e} f_{\Lambda\Sigma} F^{\Lambda\,\mu\nu} \star F^\Sigma{}_{\mu\nu} - V \right]$$

where the scalar potential $V(Z, Z^*)$ is given by

$$V(Z, Z^*) = -24|\mathcal{L}|^2 + 8\mathcal{G}^{ij^*} \mathcal{D}_i \mathcal{L} \mathcal{D}_{j^*} \mathcal{L}^* + \tfrac{1}{2} g^2 (\Im\mathrm{m}\, f)^{-1|\Lambda\Sigma} \mathcal{P}_\Lambda \mathcal{P}_\Sigma \,,$$

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Then, a bosonic configuration ($\phi^f = 0$) will be invariant under the infinitesimal supersymmetry transformation generated by the parameter $\epsilon^\alpha(x)$ if it satisfies the *Killing Spinor Equations* (KSEs) (one for each f)

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In $N = 1, d = 4$ supergravity the fermionic supersymmetry transformations are

$$\begin{aligned}
\delta_\epsilon \psi_\mu &= \mathfrak{D}_\mu \epsilon + i\mathcal{L}\gamma_\mu \epsilon^* = \left[\nabla_\mu + \tfrac{i}{2}\hat{\mathcal{Q}}_\mu\right]\epsilon + i\mathcal{L}\gamma_\mu\epsilon^* \,, \\
2\delta_\epsilon \lambda^\Lambda &= \left[\not{F}^{\Lambda +} - ig(\Im\mathrm{m}\, f)^{-1|\Lambda\Sigma}\mathcal{P}_\Sigma\right]\epsilon \,, \\
\delta_\epsilon \chi^i &= i\not{\mathfrak{D}} Z^i \epsilon^* + 2\mathcal{D}^i\mathcal{L}^* \epsilon \,.
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Our goal is to find, for all possible $N = 1, d = 4$ SUGRAs all the bosonic field configurations $e^a{}_\mu(x), A^\Lambda{}_\mu(x), Z^i(x)$ that admit Killing spinors and then impose the equations of motion to find supersymmetric solutions.

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This method does not classify the supersymmetric configurations by their number of independent Killing spinors. It should be supplemented by the spinorial geometry method of Papadopoulos, Gran, Roest, Gutowski et al.

General results

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If the superpotential $\mathcal{L} \neq 0$, $dl \neq 0$ and we are going to have domain-walls.

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- The general form of the metric is

$$ds^2 = 2V du(dv + H du + \hat{\omega}) - 2V^{2\alpha} e^{2U} dz dz^* = 2\hat{l} \otimes \hat{n} - 2\hat{m} \otimes \hat{m}^* \,.$$

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- The form of the vector field strengths is

$$F^{\Lambda\,+} = \tfrac{1}{2}\phi^{\Lambda}\hat{l} \wedge \hat{m}^* + \tfrac{i}{4} g \Im\mathrm{m} f^{-1|\Lambda\Sigma} \mathcal{P}_{\Sigma} (\hat{l} \wedge \hat{n} + \hat{m} \wedge \hat{m}^*) \,,$$

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- The functions V, H, U, ϕ^{Λ} and the 1-form $\hat{\omega}$ satisfy the 1st-order equations

$$\begin{aligned}
m^{\mu}\partial_{\mu} \log V &= -2\sqrt{2}\,\mathcal{L}^* \, , \\
\mathcal{L}^* &= -\tfrac{1}{\sqrt{2}} m^{\mu}[\partial_{\mu} \log (V^{\alpha} e^{U}) - i\hat{\mathcal{Q}}_{\mu}] \, , \\
(d\omega)_{zz^*} &= 2in^{\mu}\hat{\mathcal{Q}}_{\mu} \, .
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All these configurations admit Killing spinors which are constant (in a certain gauge) and satisfy the projection

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and, if $\mathcal{L} \neq 0$

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Many terms vanish automatically because they are odd in fermion fields ϕ^f

$$\delta_\epsilon \phi^b\big|_{\phi^f=0} = S_{,f}\big|_{\phi^f=0} = (\delta_\epsilon \phi^f)_{,f_1}\big|_{\phi^f=0} = 0\,,$$

and we get

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$$N=1, d=4: \qquad \left.\begin{array}{rcl} \delta_\epsilon e^a{}_\mu & = & -\frac{i}{4}\bar{\psi}_\mu \gamma^a \epsilon^* + \text{c.c.}\,, \\ \delta_\epsilon A^\Lambda{}_\mu & = & \frac{i}{8}\bar{\lambda}^\Lambda \gamma_\mu \epsilon^* + \text{c.c.}\,, \\ \delta_\epsilon Z^i & = & \frac{1}{4}\bar{\chi}^i \epsilon\,. \end{array}\right\}$$

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and we get

$$\left\{ S_{,b}\, (\delta_\epsilon \phi^b)_{,f_1} + S_{,f\,f_1}\, \delta_\epsilon \phi^f \right\} \Big|_{\phi^f=0} = 0\,.$$

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That is: the scalar equations of motion are always automatically satisfied for supersymmetric configurations and we only need to check the components

$$\mathcal{E}^{uu} = 0\,, \qquad \mathcal{B}^{\Lambda\, u} = 0\,, \qquad \mathcal{E}_\Lambda{}^u = 0\,.$$

Examples:

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Under investigation

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All vector fields are collectively denoted by $A^\Lambda{}_\mu = (A^0{}_\mu, A^i{}_\mu)$ and the complex scalars, which parametrize a special-Kähler manifold ($\Rightarrow$ Hodge) described by constrained symplectic sections $(\mathcal{L}^\Lambda(Z, Z^*), \mathcal{M}_\Lambda(Z, Z^*))$.

The $4n_H$ real scalars q^u parametrize a quaternionic-Kähler manifold.

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There is no superpotential in $N=2, d=4$ SUGRA and the kinetic matrix $f_{\Lambda\Sigma}(Z)$ (now called period matrix and denoted by $\mathcal{N}_{\Lambda\Sigma}(Z,Z^*)$) is not as arbitrary as in $N=1$.

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The action of the bosonic fields of the ungauged theory is

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\begin{aligned}
S \;\; = \;\; & \int d^4x \sqrt{|g|} \, \big[R + 2\mathcal{G}_{ij^*} \partial_\mu Z^i \partial^\mu Z^{*\,j^*} + 2\Im\mathrm{m}\mathcal{N}_{\Lambda\Sigma} F^{\Lambda\,\mu\nu} F^{\Sigma}{}_{\mu\nu} \\
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The symmetries of this theory that can be gauged are

- ☞ R-symmetry $SU(2) \times U(1)$.
- ☞ Isometries of $\mathcal{G}_{ij^*}(Z, Z^*)$ that are symmetries of the full theory. $N = 2$ does not admit Fayet-Iliopoulos-like terms in this sector and only non-Abelian groups can be gauged in it. This is the case that we are going to study.
- ☞ Isometries of $\mathsf{H}_{uv}(q)$ that are symmetries of the full theory. The gauging of the R-symmetry can be seen as a limiting case of this (via Fayet-Iliopoulos-like terms) and has been studied in Tod (1983) and Cacciatori, Klemm, Mansi & Zorzan, `arXiv:0804.0009 [hep-th]`.

Then, we consider $N = 2, d = 4$ SUGRA coupled to non-Abelian vector fields and with no hypers, that is: $N = 2, d = 4$ Einstein-Yang-Mills theories.

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where the potential is given by

$$V(Z, Z^*) = -\tfrac{1}{4} g^2 \Im\mathrm{m}\mathcal{N}^{-1|\Lambda\Sigma} \mathcal{P}_\Lambda \mathcal{P}_\Sigma \geq 0 .$$

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The supersymmetry transformation rules of the fermions for vanishing fermions are

$$\begin{aligned} \delta_\epsilon\psi_{I\,\mu} &= \mathfrak{D}_\mu\epsilon_I + \varepsilon_{IJ}T^{+}{}_{\mu\nu}\gamma^\nu\epsilon^J\,, \qquad \mathfrak{D}_\mu\epsilon_I \equiv \{\nabla_\mu + \tfrac{i}{2}(\mathcal{Q}_\mu + gA^\Lambda{}_\mu\mathcal{P}_\Lambda)\}\epsilon_I\,, \\ \delta_\epsilon\lambda^{Ii} &= i\,\not{\mathfrak{D}}Z^i\epsilon^I + \varepsilon^{IJ}(\not{G}^{i\,+} + \tfrac{1}{2}g\mathcal{L}^{*\Lambda}k_\Lambda{}^i)\epsilon_J\,, \end{aligned}$$

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And the Killing spinor equations are

$$\begin{aligned} \delta_\epsilon \psi_{I\,\mu} &= \mathfrak{D}_\mu \epsilon_I + \varepsilon_{IJ} T^+{}_{\mu\nu}\gamma^\nu \epsilon^J = \mathbf{0}\,, \\ \delta_\epsilon \lambda^{Ii} &= i\,\not{\mathfrak{D}} Z^i \epsilon^I + \varepsilon^{IJ}(\not{G}^{i\,+} + \tfrac{1}{2} g \mathcal{L}^{*\Lambda} k_\Lambda{}^i)\epsilon_J = \mathbf{0}\,, \end{aligned}$$

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Instead of giving the general results, I am going to focus on a particular subclass of timelike supersymmetric solutions (Hübscher, Meessen, O. & Vaulà, `arXiv:0712.1530 [hep-th]`; P. Meessen, `arXiv:0803.0684 [hep-th]`, and paper in preparation). They can be constructed as follows:

RECIPE:

☞ Find a set of Yang-Mills $A^{\Lambda}{}_{m}$ and functions $\mathcal{I}^{\Lambda}$ in flat 3-d space satisfying

$$\tfrac{1}{2}\,\epsilon_{pmn}\,F^{\Lambda}{}_{mn} \;=\; -\tfrac{1}{\sqrt{2}}\,\mathfrak{D}_{p}\mathcal{I}^{\Lambda}\,,$$

which is the Bogomol'nyi equation satisfied by known magnetic monopole solutions.

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- Use the above solution to find a solution of

$$\mathfrak{D}_m\mathfrak{D}_m\mathcal{I}_{\Lambda} \;=\; \tfrac{1}{2}g^2\;\left[f_{\Lambda(\Sigma}{}^{\Gamma}f_{\Delta)\Gamma}{}^{\Omega}\,\mathcal{I}^{\Sigma}\mathcal{I}^{\Delta}\right]\,\mathcal{I}_{\Omega}\;,$$

  so that

$$\langle\,\mathcal{I}\mid\mathfrak{D}_m\mathcal{I}\rangle = \mathcal{I}_{\Lambda}\mathfrak{D}_m\mathcal{I}^{\Lambda} - \mathcal{I}^{\Lambda}\mathfrak{D}_m\mathcal{I}_{\Lambda} = 0\;.$$

$N = 1, 2$ *Supersymmetric Solutions*

☞ Solve the stabilization equations to find $\mathcal{R}^\Lambda$ and $\mathcal{R}_\Lambda$. N.B.:

$$\begin{aligned} \mathcal{I}^\Lambda &\equiv \Im\mathrm{m}(\mathcal{L}^\Lambda / X)\,, \qquad & \mathcal{I}_\Lambda &\equiv \Im\mathrm{m}(\mathcal{M}_\Lambda / X)\,, \\ \mathcal{R}^\Lambda &\equiv \Re\mathrm{e}(\mathcal{L}^\Lambda / X)\,, \qquad & \mathcal{R}_\Lambda &\equiv \Re\mathrm{e}(\mathcal{M}_\Lambda / X)\,. \end{aligned}$$

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The scalars are, then, given by

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☞ Finally, with

$$2|X|^2 = \langle\,\mathcal{R} \mid \mathcal{I}\,\rangle^{-1}\,,$$

construct the spacetime metric

$$ds^2 = 2|X|^2 dt^2 - \frac{1}{2|X|^2} dx^m dx^m\,,$$

☞ Solve the stabilization equations to find $\mathcal{R}^\Lambda$ and $\mathcal{R}_\Lambda$. N.B.:

$$\mathcal{I}^\Lambda \equiv \Im\mathrm{m}(\mathcal{L}^\Lambda/X)\,, \qquad \mathcal{I}_\Lambda \equiv \Im\mathrm{m}(\mathcal{M}_\Lambda/X)\,,$$

$$\mathcal{R}^\Lambda \equiv \Re\mathrm{e}(\mathcal{L}^\Lambda/X)\,, \qquad \mathcal{R}_\Lambda \equiv \Re\mathrm{e}(\mathcal{M}_\Lambda/X)\,.$$

The scalars are, then, given by

$$Z^i = \frac{\mathcal{L}^i}{\mathcal{L}^0} = \frac{\mathcal{L}^i/X}{\mathcal{L}^0/X} = \frac{\mathcal{R}^i + i\mathcal{I}^i}{\mathcal{R}^0 + i\mathcal{I}^0}\,.$$

☞ Finally, with

$$2|X|^2 = \langle\, \mathcal{R} \mid \mathcal{I}\,\rangle^{-1}\,,$$

construct the spacetime metric

$$ds^2 = 2|X|^2 dt^2 - \frac{1}{2|X|^2} dx^m dx^m\,,$$

and the symplectic vector of 2-form field strengths

$$\mathcal{F} = -\sqrt{2}\,\mathfrak{D}\left(\,|X|^2\mathcal{R}\,dt\right) - \sqrt{2}\,|X|^2 \star(dt \wedge\, \mathfrak{D}\mathcal{I})\,.$$

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A 2-parameter (μ and ρ) family of solutions is given by

$$\mathcal{I}(r) = \frac{\sqrt{2}\mu}{g} \mathsf{H}_\rho(\mu r) \,, \quad \mathsf{H}_\rho(r) = \coth(r + \rho) - \frac{1}{r} \,,$$

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The two most interesting cases are $\rho = 0, \infty$.

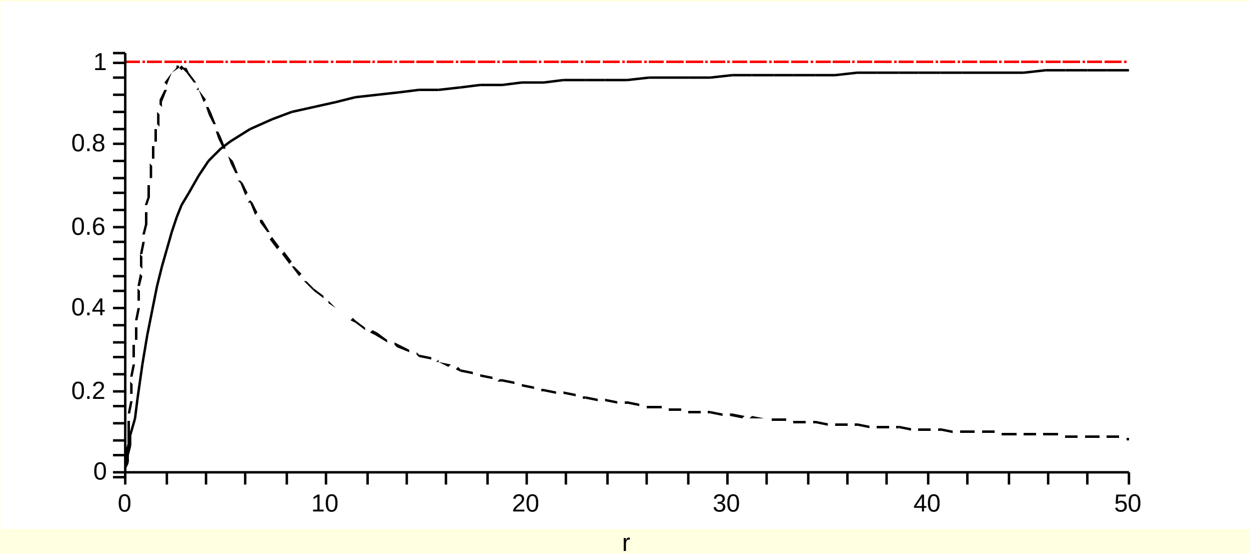
7 – 't Hooft-Polyakov Monopoles

The $\rho = 0$ solution can be written in the form

$$A^a{}_m = \varepsilon_{mb}{}^a \, n^b \, \frac{\mu}{g} \, \mathsf{G}_0(\mu r) \,, \qquad \mathsf{G}_0(r) = \frac{1}{r} - \frac{1}{\sinh r} \,,$$

$$\mathcal{I}^a = \frac{\sqrt{2}\mu}{g} \, \mathsf{H}_0(\mu r) \, n^a \,, \qquad \mathsf{H}_0(r) = \coth r - \frac{1}{r} \,.$$

The profiles of the functions G and H are



$\mathcal{I}^a$ is regular at $r = 0$ for $\rho = 0$, and describes the 't Hooft-Polyakov monopole.

8 – Black Hedgehogs

In the limit $\rho \to \infty$ we find the "black hedgehog" solution

$$\mathcal{I}^a = -\sqrt{2}\left(\mathcal{I}_\infty + \frac{1}{gr}\right) n^a \,,$$

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The YM field is singular at $r = 0$ but in EYM theory the coupling to gravity may cover it by an event horizon.

The possible existence of an event horizon covering the singularity at $r = 0$ has to be studied in specific models.

Before finding $\mathcal{R}$ and $|X|$ we have to find the $\mathcal{I}_a$s solving

$$\mathfrak{D}_m \mathfrak{D}_m \mathcal{I}_\Lambda \;=\; \tfrac{1}{2} g^2 \; \left[f_{\Lambda(\Sigma}{}^{\Gamma} f_{\Delta)\Gamma}{}^{\Omega} \, \mathcal{I}^{\Sigma} \mathcal{I}^{\Delta} \right] \, \mathcal{I}_{\Omega} \;,$$

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$$\mathcal{I}_a \;=\; \frac{g}{2} \mathcal{J} \, \mathcal{I}^a \; ,$$

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If we split the index Λ into an a-index and an u-index labeling the ungauged directions, the staticity constraint only acts non-trivially on the ungauged part:

$$\mathcal{I}_u \; d\mathcal{I}^u \;-\; \mathcal{I}^u \; d\mathcal{I}_u \;+\; \mathcal{I}_a \; \mathfrak{D}\mathcal{I}^a \;-\; \mathcal{I}^a \; \mathfrak{D}\mathcal{I}_a \;=\; \mathcal{I}_u \; d\mathcal{I}^u \;-\; \mathcal{I}^u \; d\mathcal{I}_u \;=\; 0 \;,$$

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This determines completely the family of solutions but, in order to find explicit expressions for $\mathcal{R}$ and $|X|$ and the spacetime metric we must solve the stabilization equations which depend on the specific model considered.

Metrics

For simplicity let us consider a $\overline{\mathbb{CP}}^3$ model whose prepotential reads

$$\mathcal{F} \;=\; \tfrac{i}{4}\,\eta_{\Lambda\Sigma}\;\mathcal{X}^{\Lambda}\;\mathcal{X}^{\Sigma}\,, \qquad \eta \;=\; \mathrm{diag}\,(\,-\;,\;[+]^{n}\;)\;.$$

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With the hedgehog Ansatz $\mathcal{I}^{a\,2} = \mathcal{I}^2$ and $SU(2)$ effectively reduces to a $U(1)$ in the metric! For black holes with finite entropy (attractor) we need at least two $U(1)$s. However, since $\mathcal{I}^a$ is bound in the monopole, we do not need $\mathcal{I}^0, \mathcal{I}_0$ and we can set them to constants.

Normalizing to have asymptotic flatness, we get, for the monopole

$$-g_{rr} = 1 \ + \ \mu^2 \left[\frac{1}{g^2} \ + \ \mathcal{J}^2\right] \ \left[1 \ - \ \mathsf{H}^2(\mu r)\right] \ ,$$

which is completely regular and describes an object of mass

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To embed the black hedgehog into this model and get a regular solution ($|Z|^2 < 1$) we need non-trivial $\mathcal{I}^0$ or $\mathcal{I}_0$. The conditions for regularity are the same as in an standard, Abelian $U(1) \times U(1)$ black hole of this model:

$$\mathsf{M} \;\; = \;\; \mathcal{I}^0_\infty p^0 + \mathcal{I}_{0\infty} q_0 - 2\mu \left[1/g^2 \; + \; \mathcal{J}^2\right] > 0 \, ,$$

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How does the attractor mechanism work in this solution?

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- There is still much work to do to classify all the possible supersymmetric solutions....

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THANKS!