SUSY and cosmic censorship

Tomás Ortín (I.F.T., Madrid)

Seminar given on October 9th 2006 at the 2$^{\text{nd}}$ Workshop and Midterm Meeting of the RTN Constituents, Fundamental Forces and Symmetries of the Universe

Based on hep-th/0606281 and on work in preparation. Work done in collaboration with

Jorge Bellorín and Patrick Meessen (IFT, Madrid, Spain)

Plan of the Talk:

1 – The 1992 SUSY *versus* cosmic censorship conjecture

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$$ds^2 = \frac{(r-r_+)(r-r_-)}{r^2}dt^2 - \frac{r^2}{(r-r_+)(r-r_-)}dr^2 - r^2 d\Omega^2_{(2)}\,,$$

$$r_\pm = M \pm \sqrt{M^2 - q^2}\,, \quad \Rightarrow \quad M^2 \geq q^2 \quad \text{(BPS bound)}.$$

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$\Rightarrow$ Supersymmetry works as a cosmic censor (Kallosh, Linde, O., Peet & Van Proeyen (1992)).

2 – Supersymmetric but singular solutions

The conjecture fails for the simplest black-hole-type stationary supersymmetric solutions of pure $N = 2, d = 4$ SUGRA (Perjés (1971), Israel & Wilson (1972), Tod (1983):

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ds^2 &= |V|^2(dt + \omega)^2 - |V|^{-2}d\vec{x}^2\,, \\
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- ➾ They can be removed at the expense of asymptotic flatness (Misner 1963).

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We will show that only the regular solutions which can be described by String Theory are everywhere supersymmetric).

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II (in presence of scalars) The attractor equations

$$\mathfrak{D}_i \mathcal{Z}|_{Z^i = Z^i_{\rm fix}} = 0\,.$$

must be satisfied at each of the sources for admissible values of the scalars (no hair) and the value of the central charge must be finite.

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Now let us see how these conditions select regular solutions that can be described microscopically by String Theory.

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KSIs: relation between the equations of motion of the bosonic fields $\mathcal{E}^{\mu\nu} \equiv \frac{\delta S}{\delta g_{\mu\nu}}$, $\mathcal{E}^{\mu} \equiv \frac{\delta S}{\delta A_{\mu}}$ evaluated on supersymmetric configurations.

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These are precisely the supersymmetric *sources* String Theory does not account for.

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To have solutions with angular momentum we need to add matter fields.

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There is a new KSI involving the equations of motion of the scalars $\mathcal{E}_{i^*} \equiv \frac{\delta S}{\delta Z^{*\,i^*}}$:

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This equation means that when the attractor equations are satisfied there are no scalar sources, i.e. $\mathcal{E}_{i^*} = 0$ *anywhere*. It is not known how to account for these sources in String Theory.

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\mathcal{I}^1 &= \frac{1}{\sqrt{2}} + \frac{8q}{r_1} + \frac{8q}{r_2}\,,\\
\mathcal{I}_0 &= -\frac{4q}{r_2}\,,\\
\mathcal{I}_1 &= -\frac{1}{4\sqrt{2}} - \frac{q}{r_1} + \frac{q}{r_2}\,,
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\end{aligned}
$$

where $q > 0$, leads to the metric component

$$
-g_{rr} = 1 + \frac{9\sqrt{2}q}{r_1} + \frac{10\sqrt{2}q}{r_2} + \frac{16q^2}{r_1^2} + \frac{8q^2}{r_2^2} + \frac{40q^2}{r_1 r_2}\,,
$$

which is finite everywhere outside $r_{1,2} = 0$.

An explicit example for the prepotential $\mathcal{F} = -i\mathcal{X}^0\mathcal{X}^1$: choose the four real harmonic functions

$$\begin{aligned}
\mathcal{I}^0 &= \frac{1}{\sqrt{2}} + \frac{q}{r_1} + \frac{q}{r_2}\,, \qquad r_{1,2} \equiv |\vec{x} - \vec{x}_{1,2}|\,,\\
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which is finite everywhere outside $r_{1,2} = 0$. In particular the “mass” of each of the two objects is positive

$$M_1 = 9q/\sqrt{2}\,, \qquad M_2 = 5\sqrt{2}q\,, \qquad M = M_1 + M_2 = 19q/\sqrt{2}\,,$$

In the $r_{1,2} \to 0$ limits we find spheres of finite areas

$$\frac{A_1}{4\pi} = 16q^2 = 2|\mathcal{Z}_{\text{fix},1}|^2\,, \qquad \frac{A_2}{4\pi} = 8q^2 = 2|\mathcal{Z}_{\text{fix},2}|^2\,.$$

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The total horizon area is

$$\frac{A}{4\pi} = \frac{A_1}{4\pi} + \frac{A_2}{4\pi} = 24q^2 < 2|\mathcal{Z}_{\text{fix,tot}}|^2 = 64q^2\,,$$

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To have zero *NUT* charge, we must fix

$$r_{12} \equiv |\vec{x}_2 - \vec{x}_1| = 12\sqrt{2}q \,.$$

The angular momentum is, then, finite and given by

$$|J| = 12q^2 \,.$$

6 – Preliminary results in $N = 1, d = 5$ SUGRA

KSIs different: there are regular supersymmetric rotating black holes and rings.

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The 1$^{\text{st}}$ KSI is the integrability of the ω equation

$$\mathcal{E}^{m0} + \frac{\sqrt{3}}{4} h^I \mathcal{E}_I{}^m = \frac{1}{2} f^{-5/2} [\star_4 d^2 \omega]^m\,,$$

which is satisfied by regular supersymmetric black hole (Breckenridge, Myers, Peet & Vafa (1996)) and ring (Elvang,Emparan, Mateos & Reall, (2004)) solutions.