Duality

and

Massive Supergravities

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P. Meessen & T.O., `hep-th/9806120`

1. Introduction: String dualities from the string effective action.
   Examples:
   (a) Bosonic string T duality.
   (b) 4-d S duality.
   (c) $d = 11$ SUGRA as the strong-coupling limit of the type IIA string.
   (d) Type II superstring T duality.

2. Generalized dimensional reduction.
   (a) Definition and examples.
   (b) A toy model and physical interpretation.

3. Type IIB S duality and GDR.
   (a) 9-dimensional massive type II theories.
   (b) Physical interpretation: pq-7-branes.
   (c) Eleven-dimensional origin and T duality.

Introduction/Motivation

All superstring dualities which act on the massless sector manifest themselves in the effective action. All one needs to have is a good dictionary: $(p+1)$-form potentials are associated to p-branes and scalars to *moduli* (S moduli are coupling constants). The effective field theories of compactified superstring theories can be obtained by *standard* Kaluza-Klein **dimensional reduction** (i.e. keeping only the massless zeromodes.) String dualities involving compactification can be studied through these field theories.

Example 1.
Bosonic String T Duality
(invariance under the exchange of winding and momentum modes together with the inversion of the compactification radius) corresponds to the symmetry that appears when the $\hat{d}$-dimensional string effective action:

$$\hat{S} = \int d^{\hat{d}}\hat{x}\sqrt{|\hat{g}|}\; e^{-2\hat{\phi}}\left[\hat{R} - 4(\partial\hat{\phi})^2 + \tfrac{1}{2\cdot 3!}\hat{H}^2\right],$$

is dimensionally reduced to $d = (\hat{d}-1)$ dimensions

$$\begin{aligned} S \;=\; & \int dy \int d^dx\sqrt{|g|}\; e^{-2\phi}\left[R - 4(\partial\phi)^2 \right. \\ & \left. -\tfrac{1}{4}k^2F^2(A) - \tfrac{1}{4}k^{-2}F^2(B) + \tfrac{1}{2\cdot 3!}H^2\right], \end{aligned}$$

under the involution

$$\left\{ \begin{array}{rcl} A_\mu & \leftrightarrow & B_\mu\,, \\ k & \leftrightarrow & k^{-1}\,. \end{array} \right.$$

Maharana & Schwarz,
Bergshoeff, Kallosh, & T.O.

Physical Interpretation:

Observe that $k(x) = \hat{g}_{yy}(x)$ is the local radius:

$$2\pi R(x) = \int dyk(x) = k(x) \int dy\,,$$

so the radius is inverted in T duality.

On the other hand, in the full KK/string spectrum, there are massive **particle** states charged with respect to

1.- $A_\mu = \hat{g}_{\mu y}/\hat{g}_{yy}$ (KK or momentum modes)

and with respect to

2.- $B_\mu = \hat{B}_{\mu y}$ (winding modes).

Thus, we can also see in the effective action that momentum and winding modes are interchanged.

Finally: in $\hat{d}$-dimensional language we have simply obtained *Buscher's T duality rules*.

T. Buscher

Example 2.
Four-Dimensional Superstring S Duality (strong-weak coupling duality) manifests itself in the classical $Sl(2,\mathbb{R})$ invariance of the $N = 4, d = 4$ SUGRA action (which is the effective action of the heterotic superstring compactified on T^6):

$$S = \int d^4x\sqrt{|g_E|}\ \Big[R_E + 2(\partial\phi)^2 + \tfrac{1}{2}e^{4\phi}(\partial a)^2 - \tfrac{1}{4}e^{-2\phi}F^I F^I - \tfrac{1}{4}aF^{I\star}F^I\Big]\,,$$

under

$$\lambda \to \frac{a\lambda + b}{c\lambda + d}, \quad \begin{pmatrix} \mathcal{F} \\ F \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \mathcal{F} \\ F \end{pmatrix},$$

where

$$\lambda = a + ie^{-2\phi}\,,$$

and

$$^\star\mathcal{F} = e^{-2\phi}F + a^\star F\,, \qquad ad - bc = 1\,.$$

Physical Interpretation:

e^{ϕ} is the string coupling constant (it works like a local coupling constant).

a (the pseudoscalar axion) is like a local θ-parameter (even though here we only have Abelian vector fields).

$Sl(2,\mathbb{R})$ is generated by two kinds of transformations:

$$\lambda \to -1/\lambda\,,$$

which inverts the coupling constant and

$$\lambda \to \lambda + \text{constant}\,,$$

(Peccei-Quinn-type shifts of the axion).

A. Sen

Example 3.
11-Dimensional SUGRA as the String-Coupling Limit of Type IIA Superstring Theory
(in the strong-coupling limit, type IIA superstring theory, which is a 10-dimensional theory, behaves effectively as an 11-dimensional one. The effective field theory in the strong-coupling limit can only be the unique 11-dimensional supergravity.)

This can actually be seen by performing the standard Kaluza-Klein dimensional reduction of 11-dimensional SUGRA down to $d = 10$, $N = 2A$ SUGRA (the effective field theory of the type IIA superstring) and then one can find the relation between the radius of the 11th dimension and the dilaton (the string coupling constant).

Witten

Let us introduce the bosonic sectors of 11-dimensional SUGRA and $d = 10, N = 2A$ SUGRA.

The $\hat{\hat{d}} = 11, N = 1$ **SUGRA**

$$\hat{\hat{S}} = \int d^{11}\hat{\hat{x}} \sqrt{|\hat{\hat{g}}|} \left[\hat{\hat{R}} - \frac{1}{2\cdot 4!}\hat{\hat{G}}^2 + \frac{1}{6^4}\frac{1}{\sqrt{|\hat{\hat{g}}|}}\hat{\hat{\epsilon}}\partial\hat{\hat{C}}\partial\hat{\hat{C}}\hat{\hat{C}}\right],$$

Fields: $\left\{\hat{\hat{g}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}}, \hat{\hat{C}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}}\right\}$.

Field strengths: $\hat{\hat{G}} = 4\partial\hat{\hat{C}}$.

Remark: the transformation $\hat{\hat{C}} \to -\hat{\hat{C}}$ does not leave invariant the action. There are, in fact **two** 11-dimensional SUGRAs. The difference is the sign in the topological term

Cremmer, Julia & Scherk

$\hat{d} = 10, N = 2A$ (non-chiral) SUGRA (I)

$$
\begin{aligned}
\hat{S} \;=\;& \\
&\int d^{10}\hat{x}\;\sqrt{|\hat{g}|}\left\{e^{-2\hat{\phi}}\left[\hat{R} - 4\left(\partial\hat{\phi}\right)^2 + \tfrac{1}{2\cdot 3!}\hat{H}^2\right]\right. \\
&\quad - \left[\tfrac{1}{4}\left(\hat{G}^{(2)}\right)^2 + \tfrac{1}{2\cdot 4!}\left(\hat{G}^{(4)}\right)^2\right] \\
&\quad \left. + \tfrac{1}{144}\tfrac{1}{\sqrt{|\hat{g}|}}\;\hat{\epsilon}\partial\hat{C}^{(3)}\partial\hat{C}^{(3)}\hat{B}\right\}\,.
\end{aligned}
$$

where

NS-NS fields: $\{\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{B}_{\hat{\mu}\hat{\nu}}, \hat{\phi}\}$

RR potentials: $\{\widehat{C^{(1)}}, \widehat{C^{(3)}}_{\hat{\mu}\hat{\nu}\hat{\rho}}\}$.

$\hat{d} = 10, N = 2A$ (non-chiral) SUGRA (II)

Field strengths:

$$\begin{cases} \hat{H} &= 3\partial\hat{B}\,, \\ \hat{G}^{(2)} &= 2\partial\hat{C}^{(1)}\,, \\ \hat{G}^{(4)} &= 4\left(\partial\hat{C}^{(3)} - 3\partial\hat{B}\hat{C}^{(1)}\right)\,, \end{cases}$$

Remark: there are **two** different $\hat{d} = 10, N = 2A$ SUGRAs as well. The difference is the sign in the topological term. Again, they are related but not equal.

Huq & Namazie,
Giani & Pernici,
Campbell & West,

The fact that the reduction of $\hat{\hat{d}} = 11$, $N = 1$ SUGRA to 10 dimensions gives a 10-dimensional, $N = 2$ non-chiral theory (called $N = 2A$) was well-known. However, the action had never been written in stringy variables and the relation of the stringy variables with the 11-dimensional fields was unknown. That relation is given by

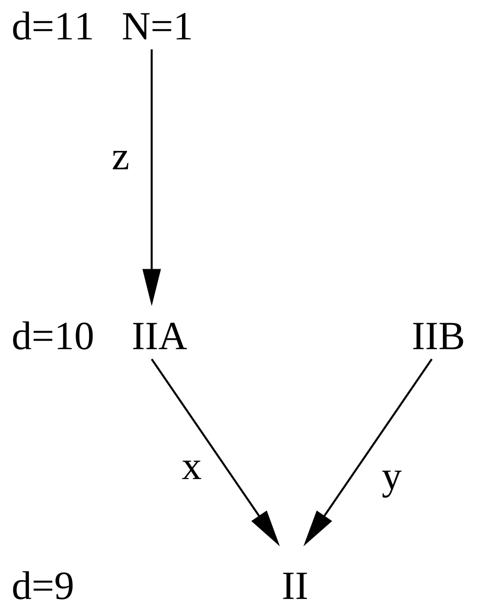
$$\left\{\begin{array}{rcl}
\hat{\hat{g}}_{\hat{\mu}\hat{\nu}} & = & e^{-\frac{2}{3}\hat{\phi}}\hat{g}_{\hat{\mu}\hat{\nu}} - e^{\frac{4}{3}\hat{\phi}}\hat{C}^{(1)}{}_{\hat{\mu}}\hat{C}^{(1)}{}_{\hat{\nu}}\,, \\
\hat{\hat{g}}_{\hat{\mu}\underline{z}} & = & -e^{\frac{4}{3}\hat{\phi}}\hat{C}^{(1)}{}_{\hat{\mu}}\,, \\
\hat{\hat{g}}_{\underline{zz}} & = & -e^{\frac{4}{3}\hat{\phi}}\,. \\
\hat{\hat{C}}_{\hat{\mu}\hat{\nu}\hat{\rho}} & = & \hat{C}^{(3)}{}_{\hat{\mu}\hat{\nu}\hat{\rho}}\,, \\
\hat{\hat{C}}_{\hat{\mu}\hat{\nu}\underline{z}} & = & \hat{B}_{\hat{\mu}\hat{\nu}}\,,
\end{array}\right.$$

The radius of the compact dimension is related to the dilaton by

$$2\pi R_z(x) = \int dz \sqrt{|g_{zz}(x)|} = e^{\frac{2}{3}\hat{\phi}(x)} \int dz\,.$$

In the strong-coupling limit $e^{\hat{\phi}}$ gets big and so does R_z. The 11th dimension becomes macroscopic.

Example 4.
Type II Superstring T duality
(the exchange of winding and momentum modes of strings and **D-branes** together with the inversion of the compactification radius takes us from type IIA to type IIB theory and vice-versa) manifests itself in the fact that the dimensional reduction of the $d = 10, N = 2A$ SUGRA action and the **T dual dimensional reduction** of the $d = 10, N = 2B$ SUGRA action give the same $d = 9, N = 2$ theory



Dai, Leigh & Polchinski, *Dine, Huet & Seiberg,* *Bergshoeff, Hull & T.O.*

Let us introduce $\hat{d} = 10, N = 2B$ SUGRA.

The $\hat{d} = 10, N = 2B$ (chiral) SUGRA "NSD Action" (I)

The equations of motion of $\hat{d} = 10, N = 2B$ supergravity can be obtained from the "non-self-dual" action

$$
\begin{aligned}
S_{\text{NSD}} \; = \\
\int d^{10}\hat{x}\; \sqrt{|\hat{\jmath}|}\; \Big\{ e^{-2\hat{\varphi}} \Big[\hat{R}(\hat{\jmath}) - 4\,(\partial\hat{\varphi})^2 + \tfrac{1}{2\cdot 3!}\hat{\mathcal{H}}^2 \Big] \\
+ \tfrac{1}{2}\left(\hat{G}^{(0)}\right)^2 + \tfrac{1}{2\cdot 3!}\left(\hat{G}^{(3)}\right)^2 + \tfrac{1}{4\cdot 3!}\left(\hat{G}^{(5)}\right)^2 \\
- \tfrac{1}{192}\tfrac{1}{\sqrt{|\hat{\jmath}|}}\;\hat{\epsilon}\;\partial\hat{C}^{(4)}\partial\hat{C}^{(2)}\hat{\mathcal{B}} \Big\}\,,
\end{aligned}
$$

where

NS-NS fields: $\{\hat{\jmath}_{\hat{\mu}\hat{\nu}}, \hat{\mathcal{B}}_{\hat{\mu}\hat{\nu}}, \hat{\varphi}\}$

RR potentials: $\{\widehat{C^{(0)}}, \widehat{C^{(2)}}_{\hat{\mu}\hat{\nu}}, \widehat{C^{(4)}}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\}$.

The $\hat{d} = 10, N = 2B$ (chiral) SUGRA "NSD Action" (II)

Field strengths:

$$\begin{cases} \hat{\mathcal{H}} &= 3\partial\hat{\mathcal{B}}\,, \\ \hat{G}^{(1)} &= \partial\hat{C}^{(0)}\,, \\ \hat{G}^{(3)} &= 3\left(\partial\hat{C}^{(2)} - \partial\hat{\mathcal{B}}\hat{C}^{(0)}\right)\,, \\ \hat{G}^{(5)} &= 5\left(\partial\hat{C}^{(4)} - 6\partial\hat{\mathcal{B}}\hat{C}^{(2)}\right)\,. \end{cases}$$

The equations of motion derived from the NSD action have to be supplemented by the self-duality condition

$$\hat{G}^{(5)} = +\,{}^{\star}\hat{G}^{(5)}\,.$$

Remark: there are **two** different $\hat{d} = 10, N = 2B$ SUGRAs, with (anti-) self-dual 5-forms and opposite chiralities. The sign of the topological term is different.

Bergshoeff, Boonstra & T.O.

$d=9, N=2$ **SUGRA (I)**

In the Einstein frame, the fields are

$$\{g_{E\ \mu\nu}, A_{(3)\ \mu\nu\rho}, \vec{A}_{(2)\ \mu\nu}, \vec{A}_{(1)\ \mu}, A_{(1)\ \mu}, K, \mathcal{M}\}\,,$$

$$
\begin{aligned}
S \;=\; & \\
& \int d^9x\ \sqrt{|g_E|}\ \Big\{R_E + \tfrac{9}{14}\,(\partial \log K)^2 + \tfrac{1}{4}\mathrm{Tr}\left(\partial\mathcal{M}\mathcal{M}^{-1}\right)^2 \\
& \qquad -\tfrac{1}{4}K^{-12/7}F^2_{(2)} - \tfrac{1}{4}K^{\frac{9}{7}}\vec{F}^{\,T}_{(2)}\mathcal{M}^{-1}\vec{F}_{(2)} \\
& \qquad +\tfrac{1}{2\cdot 3!}K^{-3/7}\vec{F}^{\,T}_{(3)}\mathcal{M}^{-1}\vec{F}_{(3)} - \tfrac{1}{2\cdot 4!}K^{6/7}F^2_{(4)} \\
& -\tfrac{1}{2^7\cdot 3^2}\tfrac{1}{\sqrt{|g_E|}}\ \epsilon\Big\{16(\partial A_{(3)})^2A_{(1)} - 24\partial A_{(3)}\partial\vec{A}^{\,T}_{(2)}\eta\vec{A}_{(2)} \\
& -24\partial A_{(3)}\left(4\vec{A}^{\,T}_{(2)}\eta\partial\vec{A}_{(1)} + 2\vec{A}^{\,T}_{(1)}\eta\partial\vec{A}_{(2)}\right)A_{(1)} \\
& +36\left(\vec{A}^{\,T}_{(2)}\eta\partial\vec{A}_{(1)} + \vec{A}^{\,T}_{(1)}\eta\partial\vec{A}_{(2)}\right)\partial\vec{A}^{\,T}_{(2)}\eta\vec{A}_{(2)} \\
& +36\left(\vec{A}^{\,T}_{(2)}\eta\partial\vec{A}_{(1)} - \vec{A}^{\,T}_{(1)}\eta\partial\vec{A}_{(2)}\right)^2A_{(1)}\Big\}\Big\}\,.
\end{aligned}
$$

The field strengths are given by

$$\left\{\begin{array}{lcl} F_{(2)} & = & 2\partial A_{(1)}\,, \\ \vec{F}_{(2)} & = & 2\partial \vec{A}_{(1)}\,, \\ \vec{F}_{(3)} & = & 3\partial \vec{A}_{(2)} - 3A_{(1)}\vec{F}_{(2)}\,, \\ F_{(4)} & = & 4\partial A_{(3)} - 3\vec{A}^{\,T}_{(2)}\ \eta\ \vec{F}_{(2)} \\ & & +2\vec{A}^{\,T}_{(1)}\ \eta\ \vec{F}_{(3)} - 6A_{(1)}\vec{A}^{\,T}_{(1)}\ \eta\ \vec{F}_{(2)}\,, \end{array}\right.$$

Remark: yet again, there are **two** different $\hat{d} = 9, N = 2$ SUGRAs, with topological terms different by a sign. The difference is the sign in the topological term. Both signs are necessary to accommodate the two $N = 2A$ and $N = 2B$ theories.

$d = 9, N = 2$ SUGRA can be obtained from $\hat{d} = 10, N = 2B$ SUGRA by dimensional reduction using the identifications

$$
\begin{aligned}
\mathcal{M} &= e^{\hat{\varphi}} \begin{pmatrix} |\hat{\lambda}|^2 & \hat{C}^{(0)} \\ \hat{C}^{(0)} & 1 \end{pmatrix}, \\
K &= e^{\hat{\varphi}/3} |\hat{\jmath}_{\underline{yy}}|^{-2/3}, \\
A_{(1)\ \mu} &= \hat{\jmath}_{\mu\underline{y}} / \hat{\jmath}_{\underline{yy}}, \\
\vec{A}_{(1)\ \mu} &= \begin{pmatrix} \hat{C}^{(2)}{}_{\mu\underline{y}} \\ \hat{\mathcal{B}}_{\mu\underline{y}} \end{pmatrix}, \\
\vec{A}_{(2)\ \mu\nu} &= \begin{pmatrix} \hat{C}^{(2)}{}_{\mu\nu} \\ \hat{\mathcal{B}}_{\mu\nu} \end{pmatrix}, \\
A_{(3)\ \mu\nu\rho} &= \hat{C}^{(4)}{}_{\mu\nu\rho\underline{y}} - \tfrac{3}{2} \hat{\mathcal{B}}_{[\mu\nu} \hat{C}^{(2)}{}_{\rho]\underline{y}} - \tfrac{3}{2} \hat{\mathcal{B}}_{[\mu|\underline{y}|} \hat{C}^{(2)}{}_{\nu\rho]}, \\
g_{E\ \mu\nu} &= e^{-4\hat{\varphi}/7} |\hat{\jmath}_{\underline{yy}}|^{1/7} \left[\hat{\jmath}_{\mu\nu} - \hat{\jmath}_{\mu\underline{y}} \hat{\jmath}_{\nu\underline{y}} / \hat{\jmath}_{\underline{yy}} \right].
\end{aligned}
$$

$d = 9, N = 2$ SUGRA can also be obtained from $\hat{d} = 10, N = 2A$ SUGRA by dimensional reduction using the identifications

$$
\begin{aligned}
\mathcal{M} &= \\
& e^{\hat{\phi}} |\hat{g}_{\underline{xx}}|^{-1/2} \begin{pmatrix} e^{-2\hat{\phi}} |\hat{g}_{\underline{xx}}| + (\hat{C}^{(1)}{}_{\underline{x}})^2 & \hat{C}^{(1)}{}_{\underline{x}} \\ \hat{C}^{(1)}{}_{\underline{x}} & 1 \end{pmatrix}, \\
K &= e^{\hat{\phi}/3} |\hat{g}_{\underline{xx}}|^{1/2}, \\
A_{(1)\ \mu} &= -\hat{B}_{\mu\underline{x}}, \\
\vec{A}_{(1)\ \mu} &= \begin{pmatrix} \hat{C}^{(1)}{}_{\mu} - \hat{C}^{(1)}{}_{\underline{x}} \hat{g}_{\mu\underline{x}} / \hat{g}_{\underline{xx}} \\ -\hat{g}_{\mu\underline{x}} / \hat{g}_{\underline{xx}} \end{pmatrix}, \\
\vec{A}_{(2)\ \mu\nu} &= \\
& \begin{pmatrix} \hat{C}^{(3)}{}_{\mu\nu\underline{x}} + 2\hat{B}_{[\mu|\underline{x}|} \hat{C}^{(1)}{}_{\nu]} - 2\hat{C}^{(1)}{}_{\underline{x}} \hat{B}_{\mu|\underline{x}|} \hat{g}_{\nu]\underline{x}} / \hat{g}_{\underline{xx}} \\ \hat{B}_{\mu\nu} - 2\hat{B}_{[\mu|\underline{x}|} \hat{g}_{\nu]\underline{x}} / \hat{g}_{\underline{xx}} \end{pmatrix}, \\
A_{(3)\ \mu\nu\rho} &= \hat{C}^{(3)}{}_{\mu\nu\rho} - \tfrac{3}{2} \hat{g}_{[\mu|\underline{x}|} \hat{C}^{(3)}{}_{\nu\rho]\underline{x}} / \hat{g}_{\underline{xx}} \\
& \quad - \tfrac{3}{2} \hat{C}^{(1)}{}_{\underline{x}} \hat{g}_{[\mu|\underline{x}|} \hat{B}_{\nu\rho} / \hat{g}_{\underline{xx}} - \tfrac{3}{2} \hat{C}^{(1)}{}_{[\mu} \hat{B}_{\nu\rho]}, \\
g_{E\ \mu\nu} &= e^{-4\hat{\phi}/7} |\hat{g}_{\underline{xx}}|^{1/7} \left[\hat{g}_{\mu\nu} - \hat{g}_{\mu\underline{x}} \hat{g}_{\nu\underline{x}} / \hat{g}_{\underline{xx}} \right].
\end{aligned}
$$

Comparing both sets of relations one finds the type II generalization of Buscher's T duality rules.

<u>From IIA to IIB:</u>

<u>**NSNS fields**</u>

$$
\begin{array}{rclrcl}
\hat{\jmath}_{\mu\nu} & = & \hat{g}_{\mu\nu} - \left(\hat{g}_{\mu\underline{x}}\hat{g}_{\nu\underline{x}} - \hat{B}_{\mu\underline{x}}\hat{B}_{\nu\underline{x}}\right)/\hat{g}_{\underline{xx}}\,, & \hat{\jmath}_{\mu\underline{y}} & = & -\hat{B}_{\mu\underline{x}}/\hat{g}_{\underline{xx}}\,, \\
\hat{\mathcal{B}}_{\mu\nu} & = & \hat{B}_{\mu\nu} + 2\hat{g}_{[\mu|\underline{x}}\hat{B}_{\nu]\underline{x}}/\hat{g}_{\underline{xx}}\,, & \hat{\mathcal{B}}_{\mu\underline{y}} & = & -\hat{g}_{\mu\underline{x}}/\hat{g}_{\underline{xx}}\,, \\
\hat{\varphi} & = & \hat{\phi} - \frac{1}{2}\log|\hat{g}_{\underline{xx}}|\,, & \hat{\jmath}_{\underline{yy}} & = & 1/\hat{g}_{\underline{xx}}\,,
\end{array}
$$

<u>RR forms</u>

$$
\begin{array}{rcl}
\hat{C}^{(2n)}{}_{\mu_1\cdots\mu_{2n}} & = & \hat{C}^{(2n+1)}{}_{\mu_1\cdots\mu_{2n}\underline{x}} + 2n\hat{B}_{[\mu_1|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_2\cdots\mu_{2n}]} \\
 & & -2n(2n-1)\hat{B}_{[\mu_1|\underline{x}|}\hat{g}_{\mu_2|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_3\cdots\mu_{2n}]\underline{x}}/\hat{g}_{\underline{xx}}\,, \\
\hat{C}^{(2n)}{}_{\mu_1\cdots\mu_{2n-1}\underline{y}} & = & \hat{C}^{(2n-1)}{}_{\mu_1\cdots\mu_{2n-1}} \\
 & & -(2n-1)\hat{g}_{[\mu_1|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_2\cdots\mu_{2n-1}]\underline{x}}/\hat{g}_{\underline{xx}}\,.
\end{array}
$$

From IIB to IIA:

NSNS fields

$$\hat{g}_{\mu\nu} = \hat{\jmath}_{\mu\nu} - \left(\hat{\jmath}_{\mu\underline{y}}\hat{\jmath}_{\nu\underline{y}} - \hat{\mathcal{B}}_{\mu\underline{y}}\hat{\mathcal{B}}_{\nu\underline{y}}\right)/\hat{\jmath}_{\underline{yy}}\,, \qquad \hat{g}_{\mu\underline{x}} = -\hat{\mathcal{B}}_{\mu\underline{y}}/\hat{\jmath}_{\underline{yy}}\,,$$

$$\hat{B}_{\mu\nu} = \hat{\mathcal{B}}_{\mu\nu} + 2\hat{\jmath}_{[\mu|\underline{y}|}\hat{\mathcal{B}}_{\nu]\underline{y}}/\hat{\jmath}_{\underline{yy}}\,, \qquad \hat{B}_{\mu\underline{x}} = -\hat{\jmath}_{\mu\underline{y}}/\hat{\jmath}_{\underline{yy}}\,,$$

$$\hat{\phi} = \hat{\varphi} - \tfrac{1}{2}\log|\hat{\jmath}_{\underline{yy}}|\,, \qquad \hat{g}_{\underline{xx}} = 1/\hat{\jmath}_{\underline{yy}}\,,$$

RR forms

$$\begin{aligned}
\hat{C}^{(2n+1)}{}_{\mu_1\ldots\mu_{2n+1}} &= \hat{\mathcal{C}}^{(2n+2)}{}_{\mu_1\ldots\mu_{2n+1}\underline{y}} \\
&\quad -(2n+1)\hat{\mathcal{B}}_{[\mu_1|\underline{y}|}\hat{\mathcal{C}}^{(2n)}{}_{\mu_2\ldots\mu_{2n+1}]} \\
&\quad -2n(2n+1)\hat{\mathcal{B}}_{[\mu_1|\underline{y}|}\hat{\jmath}_{\mu_2|\underline{y}}\hat{\mathcal{C}}^{(2n)}{}_{\mu_3\ldots\mu_{2n+1}]\underline{y}}/\hat{\jmath}_{\underline{yy}}\,, \\
\hat{C}^{(2n+1)}{}_{\mu_1\ldots\mu_{2n}\underline{x}} &= \hat{\mathcal{C}}^{(2n)}{}_{\mu_1\ldots\mu_{2n}} \\
&\quad +2n\hat{\jmath}_{[\mu_1|\underline{y}|}\hat{\mathcal{C}}^{(2n)}{}_{\mu_2\ldots\mu_{2n}]\underline{y}}/\hat{\jmath}_{\underline{yy}}\,.
\end{aligned}$$

Bergshoeff, Hull & T.O.

Physical Interpretation:

- The NSNS sector rules are the same as in the bosonic case: KK momentum modes and winding modes are also interchanged here together with the simultaneous inversion of the compactification radius.

- States charged with respect to the RR $(p+1)$-form potentials $\widehat{C}^{(p+1)}$ (D-p-branes) are interchanged

$$\widehat{C}^{(p+1)}_{\cdots\underline{x}} \leftrightarrow \widehat{C}^{(p)}_{\cdots}\,.$$

D-p-Branes wrapping around the compact dimension are transformed into D-(p-1)-branes and vice-versa.

Polchinski

- The type IIB $Sl(2,\mathbb{R})$ σ-model scalars are the moduli of the local internal torus on the type IIA side. As such, they should be identified when they are related by $Sl(2,\mathbb{Z})$ transformations (the modular transformations of the torus).

Understanding GDR: A Toy Model

In standard Kaluza-Klein theory in the vacuum $\mathbb{R}\times S^1$ ($\hat{x}^\mu = (x^\mu, z)$) only single-valued field configurations are considered. They can be Fourier-expanded in the compact coordinate $z \sim z + 2\pi\ell$:

$$\hat{\phi}(\hat{x}) = \sum_{n\in\mathbb{Z}} e^{2\pi nz/\ell}\ \hat{\phi}^{(n)}(x)\,.$$

Then, only the massless $\hat{\phi}^{(0)}(x) \equiv \phi(x)$ is kept.

However, some fields can also be **multivalued** if the different values are related by a gauge transformation or other kinds of identifications. For example, if the scalar $\hat{\phi} \equiv \hat{\phi} + 2\pi m$, then possible field configurations fall into different **topological sectors** labeled by $N \in \mathbb{Z}$

$$\hat{\phi}^{(N)}(\hat{x}) = \frac{mNz}{\ell} + \sum_{n\in\mathbb{Z}} e^{2\pi nz/\ell}\ \hat{\phi}^{(n)}(x)\,.$$

A field like $\hat{\phi}$ living on S^1 (an **axion**) can only appear through $\partial\hat{\phi}$ so the lower-dimensional theory does not depend on z if only $\frac{mNz}{\ell} + \hat{\phi}^{(0)}(x)$ is kept.

Remarks:

1. The action for $\hat{\phi}$ is necessarily invariant under constant shifts of $\hat{\phi}$ but but not any invariant action corresponds to a scalar living in S^1.

2. In the $N \neq 0$ sectors the zeromode $\phi = \hat{\phi}^{(0)}$ transforms under$\delta z = -\Lambda(x)$ by shifts

$$\delta\phi = \tfrac{mN}{\ell}\Lambda\,,$$

   called **massive gauge transformations**. This means that it can be gauged away i.e. it is "eaten" by the KK vector which becomes massive. Gauge invariance is spontaneously broken.

3. Mathematically, for $N \neq 0$, each $\hat{\phi}^{(N)}$ is a section of a fiber-bundle with fiber S^1 and the $\hat{d}$-dimensional space as base-manifold.

4. Each sector is characterized by the topological charge

$$N = \lim_{x\to\infty} \frac{1}{2\pi\ell m} \oint d\hat{\phi}\,,$$

   which is the **winding number**.

5. A choice of topological sector is a choice of vacuum. In particular, there must be a solution with $\hat{\phi} = mNz/\ell$. We are going to see that we can understand the new vacuum as one containing a $(\hat{d}-3)$-brane in $\hat{d}$ dimensions which can be represented by that solution.

Let us consider the simple model

$$\hat{S} = \int d^{\hat{d}}\hat{x}\sqrt{|\hat{g}|}\ \left[\hat{R} + \tfrac{1}{2}\left(\partial\hat{\phi}\right)^2\right] .$$

We make the standard KK ansatz for the metric (zero-modes) but we use for the scalar $\hat{\phi}(x,z) = \phi(x) + mz$. (The rule of thumb to find the right ansatz is: if the action is invariant under constant shifts then make a shift linear in z). The result is

$$\begin{aligned} S &= \int d^dx\sqrt{|g|}\ k\ \Big[R - \tfrac{1}{4}k^2F^2_{(2)} + \tfrac{1}{2}\left(\mathcal{D}\phi\right)^2 \\ &\qquad -\tfrac{1}{2}m^2k^{-2}\Big] , \end{aligned}$$

where the field strengths are defined by

$$\left\{ \begin{array}{rcl} F_{(2)\ \mu\nu} &=& 2\partial_{[\mu}A_{(1)\ \nu]} , \\ \mathcal{D}_\mu\phi &=& \partial_\mu\phi - mA_{(1)\ \mu} . \end{array} \right. \qquad \left\{ \begin{array}{rcl} \delta z &=& -\chi , \\ \delta\phi &=& m\chi , \\ \delta A_{(1)\ \mu} &=& \partial_\mu\chi . \end{array} \right.$$

ϕ is a Stueckelberg field for $A_{(1)\ \mu}$, which becomes massive by "eating" it. We could have Hodge-dualized the scalar into a $(\hat{d}-3)$-form potential before reducing. Then the **standard KK reduction** of the equivalent model

$$\tilde{\hat{S}} = \int d^{\hat{d}}x\sqrt{|\hat{g}|}\ \left[\hat{R} + \tfrac{(-1)^{(\hat{d}-2)}}{2\cdot(\hat{d}-1)!}\hat{F}^2_{(\hat{d}-1)}\right] .$$

gives

$$\begin{aligned} \tilde{S} &= \int d^{\hat{d}-1}x\sqrt{|g|}\ k\ \Big[R - \tfrac{1}{4}k^2F^2_{(2)} + \tfrac{(-1)^{(\hat{d}-2)}}{2\cdot(\hat{d}-1)!}F^2_{(\hat{d}-1)} \\ &\qquad +\tfrac{(-1)^{(\hat{d}-3)}}{2\cdot(\hat{d}-2)!}k^{-2}F^2_{(\hat{d}-2)}\Big] , \end{aligned}$$

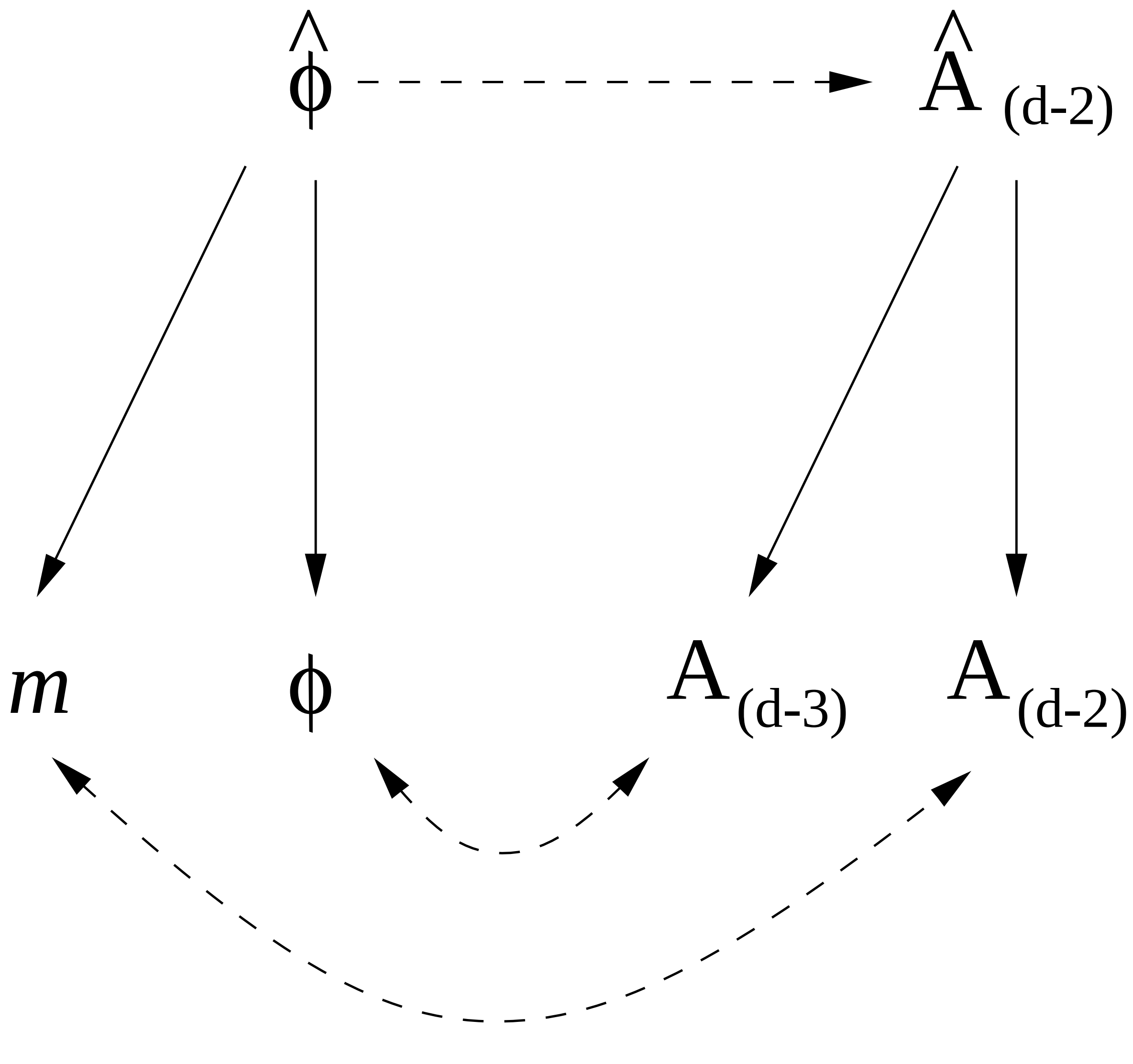
where

$$\left\{ \begin{array}{lcl} F_{(\hat{d}-1)} & = & (\hat{d}-1)\partial A_{(\hat{d}-2)} + (-1)^{(\hat{d}-1)} A_{(1)} F_{(\hat{d}-2)}\,, \\ F_{(\hat{d}-2)} & = & (\hat{d}-2)\partial A_{(\hat{d}-3)}\,, \end{array} \right.$$

are the field strengths of the $(\hat{d}-2)$- and $(\hat{d}-3)$-form potentials of the $(\hat{d}-1)$-dimensional theory. Dualizing the two potentials one gets one scalar, ϕ and one **constant** m.

We get the same theory as via GDR.

The mass parameter can be understood as the dual field strength of a $(d-1)$-form potential which couples to $(d-2)$-branes (**domain walls**). These objects do not carry any continuous degrees of freedom. They only carry topological degrees of freedom. Either you have them on your background or you don't, and that property, together with the classical solution, defines a vacuum. **Remark:** AdS space can be seen as a domain-wall solution and determines a vacuum.


φ̂
Â (d-2)
m
φ
A(d-3)
A(d-2)

S Duality in Type IIB Theory

$\hat{d}=10, N=2B$ SUGRA has an S duality global symmetry. It only becomes manifest in the Einstein frame. We made the following field-redefinitions

$$\left\{\begin{array}{rcl}\hat{\jmath}_{E\ \hat{\mu}\hat{\nu}} & = & e^{-\hat{\varphi}/2}\hat{\jmath}_{\hat{\mu}\hat{\nu}}\,, \\ \hat{\vec{\mathcal{B}}} & = & \left(\begin{array}{c}\hat{C}^{(2)} \\ \hat{\mathcal{B}}\end{array}\right), \\ \hat{D} & = & \hat{C}^{(4)}-3\hat{\mathcal{B}}\hat{C}^{(2)}\,, \\ \hat{\mathcal{M}} & = & e^{\hat{\varphi}}\left(\begin{array}{cc}|\hat{\lambda}|^2 & \hat{C}^{(0)} \\ \hat{C}^{(0)} & 1\end{array}\right), \\ \hat{\lambda} & = & \hat{C}^{(0)}+ie^{-\hat{\varphi}}\,,\end{array}\right.$$

and arrive at the following NSD action

$$\begin{array}{l}\hat{S}_{\mathrm{NSD}} \;\; = \\ \int d^{10}\hat{x}\ \sqrt{|\hat{\jmath}_E|}\left[\hat{R}(\hat{\jmath}_E)+\frac{1}{4}\mathrm{Tr}\left(\partial\hat{\mathcal{M}}\hat{\mathcal{M}}^{-1}\right)^2\right. \\ \left.+\frac{1}{2\cdot 3!}\hat{\vec{\mathcal{H}}}^{\ T}\hat{\mathcal{M}}^{-1}\hat{\vec{\mathcal{H}}}+\frac{1}{4\cdot 3!}\hat{F}^2-\frac{1}{2^7\cdot 3^3}\frac{\epsilon}{\sqrt{|\hat{\jmath}_E|}}\ \hat{D}\hat{\vec{\mathcal{H}}}^T\eta\hat{\vec{\mathcal{H}}}\right],\end{array}$$

where $\eta_{ij} = -\epsilon_{ij}$, $\hat{\vec{\mathcal{H}}} = 3\partial\hat{\vec{\mathcal{B}}}$ and we have to impose the self-duality of the 5-form field strength.

The new fields transform under $\Lambda \in Sl(2,\mathbb{R})$ according to

$$\left\{ \begin{array}{rcl} \hat{\mathcal{M}}' & = & \Lambda\hat{\mathcal{M}}\Lambda^T\,, \\ \\ \hat{\vec{\mathcal{B}}}' & = & \Lambda\hat{\vec{\mathcal{B}}}\,. \end{array} \right.$$

The remaining fields are inert and the action and self-duality constraint are invariant.

The rotation of 2-forms corresponds to the rotation of NSNS ("q") strings into RR ("p") strings and similarly for 5-branes.

Via T duality $\mathcal{M}$ can be identified with the moduli of the torus on which 11-dimensional SUGRA is compactified. Values of the moduli related by $Sl(2,\mathbb{Z})$ transformations must be considered equivalent.

Now, **if** we identify two field configurations related by $Sl(2,\mathbb{Z})$ transformations we can have different topological sectors in the compactification: "multivalued" field configurations such that going around the compact dimension once lead you to the same configuration **up to an $Sl(2,\mathbb{Z})$ transformation**. Each topological sector is defined by one $Sl(2,\mathbb{Z})$ transformation (the "monodromy" around it).

GDR is, therefore, possible. Let us do it explicitly.

GDR w.r.t. S Duality

Following the recipe, the make the following ansatz:

$$\begin{cases} \hat{\mathcal{M}}(\hat{x}) & \equiv & \Lambda(y)\hat{\mathcal{M}}^{\mathrm{b}}(x)\Lambda^T(y)\,, \\ \hat{\vec{\mathcal{B}}}(\hat{x}) & \equiv & \Lambda(y)\vec{\mathcal{B}}^{\mathrm{b}}(x)\,, \\ \hat{D}(\hat{x}) & = & \hat{D}^{\mathrm{b}}(x)\,, \end{cases}$$

where we have denoted by a superscript ^{b} the *bare* y-independent fields and where $\Lambda(y)$ is a y-dependent $Sl(2,\mathbb{Z})$ transformation. The y-dependence is better defined in the continuum $Sl(2,\mathbb{R})$ case. If $\{T_i\}$ are the generators

$$\Lambda(y) = \exp\{\tfrac{1}{2}ym^iT_i\}\,.$$

The three real parameters m^i fully determine $\Lambda(y)$ and therefore the particular compactification. These parameters are going to become masses in the lower-dimensional theory. We define the *mass matrix* $m = m^iT_i \in sl(2,\mathbb{R})$ which transforms in the adjoint

$$m' = \Lambda m \Lambda^{-1}\,,$$

and thus the three m^i transform as a triplet.

The monodromies of the fields are given by $M = e^{m/2}$:

$$\begin{cases} \hat{\mathcal{M}}(x,y+1) & = & M\hat{\mathcal{M}}(x,y)M^T\,, \\ \hat{\vec{\mathcal{B}}}(x,y+1) & = & M\hat{\vec{\mathcal{B}}}(x,y)\,. \end{cases}$$

We can now reduce. We find a theory with the same fields but new mass terms and couplings.

Massive $d = 9, N = 2$ SUGRA

Apart from some mass terms in the topological term, the theory is characterized by the new field strengths:

$$\left\{\begin{array}{rcl} \mathcal{D}\mathcal{M} & = & \partial\mathcal{M} - \left(m\mathcal{M} + \mathcal{M}m^T\right) A_{(1)}\,, \\ F_{(2)} & = & 2\partial A_{(1)}\,, \\ \vec{F}_{(2)} & = & 2\partial \vec{A}_{(1)} + m\vec{A}_{(2)}\,, \\ \vec{F}_{(3)} & = & 3\partial \vec{A}_{(2)} - 3A_{(1)}\vec{F}_{(2)}\,, \\ F_{(4)} & = & 4\partial A_{(3)} - 3\vec{A}^{\,T}_{(2)}\ \eta\ \vec{F}_{(2)} + 2\vec{A}^{\,T}_{(1)}\ \eta\ \vec{F}_{(3)} \\ & & -6A_{(1)}\vec{A}^{\,T}_{(1)}\ \eta\ \vec{F}_{(2)}\,, \end{array}\right.$$

the presence of a potential

$$\mathcal{V}\left(\mathcal{M}\right) = \tfrac{1}{2}\mathrm{Tr}\left(m^2 + m\mathcal{M}m^T\mathcal{M}^{-1}\right)\,,$$

and the invariance under the following massive gauge transformations

$$\left\{\begin{array}{rcl} \delta y & = & -\chi\,, \\ \mathcal{M}' & = & e^{\chi m}\mathcal{M}\ e^{\chi m^T}\,, \\ A'_{(1)} & = & A_{(1)} + \partial\chi\,, \\ \vec{A}'_{(1)} & = & e^{\chi m}\vec{A}_{(1)}\,, \\ \vec{A}'_{(2)} & = & e^{\chi m}\left(\vec{A}_{(2)} - 2\partial\chi\vec{A}_{(1)}\right)\,, \end{array}\right.$$

Physical Interpretation: pq-7-branes

A mass matrix m giving rise to a monodromy matrix $M = e^{m/2} \in Sl(2,\mathbb{Z})$ characterizes the vacuum. We have seen that in the GDR context the vacuum can be seen as containing $(\hat{d}-3) =$ 7-branes. Let us identify them.

First, we have D-7-branes

$$\left\{\begin{array}{rcl} ds_s^2 & = & H_{D7}^{-1/2}\left[dt^2 - d\vec{y}_7{}^2\right] - H_{D7}^{1/2} d\vec{x}_2{}^2\,, \\ e^{-2(\hat{\varphi}-\varphi_0)} & = & H_{D7}^2\,, \\ \hat{C}^{(8)}{}_{ty^1\cdots y^7} & = & \pm e^{-\hat{\varphi}_0} H_{D7}^{-1}\,, \end{array}\right.$$

and for a single D-7-brane

$$H_{D7} = h_{D7}\log|\vec{x}_2|\,.$$

In terms of the complex scalar $\hat{\lambda}$

$$\hat{\lambda} = \left\{\begin{array}{l} ie^{-\hat{\varphi}_0} h_{D7}\log\omega\,, \\ ie^{-\hat{\varphi}_0} h_{D7}\log\overline{\omega}\,, \end{array}\right. \qquad \omega = x_2^1 + ix_2^2\,,$$

The charge of a D-7-brane is just

$$\begin{array}{rcl} p & = & \oint_\gamma {}^\star\hat{G}^{(9)} = \oint_\gamma \hat{G}^{(1)} = \oint_\gamma d\hat{C}^{(0)} = \Re\mathrm{e}\oint_\gamma d\hat{\lambda} \\ & = & \mp 2\pi e^{-\hat{\varphi}_0} h_{D7}\,, \end{array}$$

If we take the unit charge D-7-brane

$$\hat{\lambda}_{(p=1)} = -\frac{1}{2\pi i} \log \overline{\omega}\,,$$

and travel once along the path $\gamma(\xi)$, $\xi \in [0,1]$, around the origin

$$\begin{aligned}
\hat{\lambda}_{(p=1)}[\gamma(1)] &= \hat{\lambda}_{(p=1)}[\gamma(0)] + 1 = \left(M_{(p=1)}\hat{\lambda}_{(p=1)}\right)[\gamma(0)]\,, \\
M_{(p=1)} &= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = T\,,
\end{aligned}$$

where $M_{(p=1)}$ is the $SL(2,\mathbb{Z})$ monodromy matrix characterizing the 7-brane with charge $p = 1$.

To define the charges in more general cases we look to the scalar equations of motion

$$\nabla_\mu \mathcal{J}^\mu = 0\,, \qquad \mathcal{J}_\mu = 2\partial_\mu \mathcal{M}\mathcal{M}^{-1}\,,$$

which are the conservation of 2 independent currents. We can define the charge matrix $\mathcal{Q}$ by

$$\mathcal{Q} \equiv \tfrac{1}{2}\oint_{S^1} \mathcal{J} = \oint_{S^1} d\mathcal{M}\mathcal{M}^{-1}\,.$$

It transforms in the adjoint representation. It is easy to see that a configuration with charge $\mathcal{Q}$ also has a monodromy matrix (also in the compact dimension case)

$$M = e^{\mathcal{Q}/2}\,.$$

The unit charge D-7-brane has the charge matrix

$$\mathcal{Q} = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}.$$

We can generate a new kind of 7-brane, "Q-7-brane" by applying $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ to the D-7-brane. It has charge matrix $\mathcal{Q} = \begin{pmatrix} 0 & -2 \\ 0 & 0 \end{pmatrix}$ and it has the form

$$\left\{ \begin{array}{rcl} d\hat{s}^2_{IIB} & = & \left(H^2_{D7} + A^2\right)^{1/2} \left[H_{D7}^{-1/2} \left(\eta_{ij} dy^i dy^j - dy^2\right) \right. \\ & & \left. -H_{D7}^{1/2} d\omega d\overline{\omega}\right] \,, \\ \hat{\lambda} & = & -1/(-A + iH_{D7}) \,, \\ H_{D7} & = & \frac{1}{4\pi} \log \omega\overline{\omega} \,, \\ A & = & \frac{1}{4\pi} i \log \omega/\overline{\omega} \,. \end{array} \right.$$

The T transformation generates out of the D-7-brane a configuration with a different constant value for $\hat{C}^{(0)}$. Although this is the only difference with the original D-7-brane solution, this constant value induces q-charge through the Witten effect. The presence of both p and q charges induces r-charge which here seems not to be independent.

Eleven-Dimensional Origin and T Duality

With only $m^3 \neq 0$, it is known that the massive 9-dimensional theory can be obtained from Romans' 10-dimensional massive $N = 2A$ theory.

Bergshoeff, de Roo, Green, Papadopoulos & Townsend

This cannot be derived from the standard 11-dimensional SUGRA. It can be derived from a non-covariant generalization in which an isometric direction is singled out. This theory could be interpreted as one in which there is a "KK9-brane" in he background. Such an object would be similar to the KK-monopole in having an isometric, compact, direction and would be described by a gauged σ-model.

Bergshoeff, Janssen & T.O. *Bergshoeff, Lozano & T.O.* *Hull*

Taking into account that we have obtained an $Sl(2, \mathbb{Z})$-covariant family of massive 9-dimensional SUGRA theories, it is clear that there must be 2 KK9-branes in the background singling out 2 isometric directions. $Sl(2, \mathbb{Z})$ rotates them in internal space.

Reducing in one of those directions one gets a generalization of Romans' theory with one KK8-brane singling one one direction.

Let us show the "massive" 11-dimensional SUGRA theory.

“Massive” 11-dimensional SUGRA

$$\hat{\hat{S}} \;=\;$$

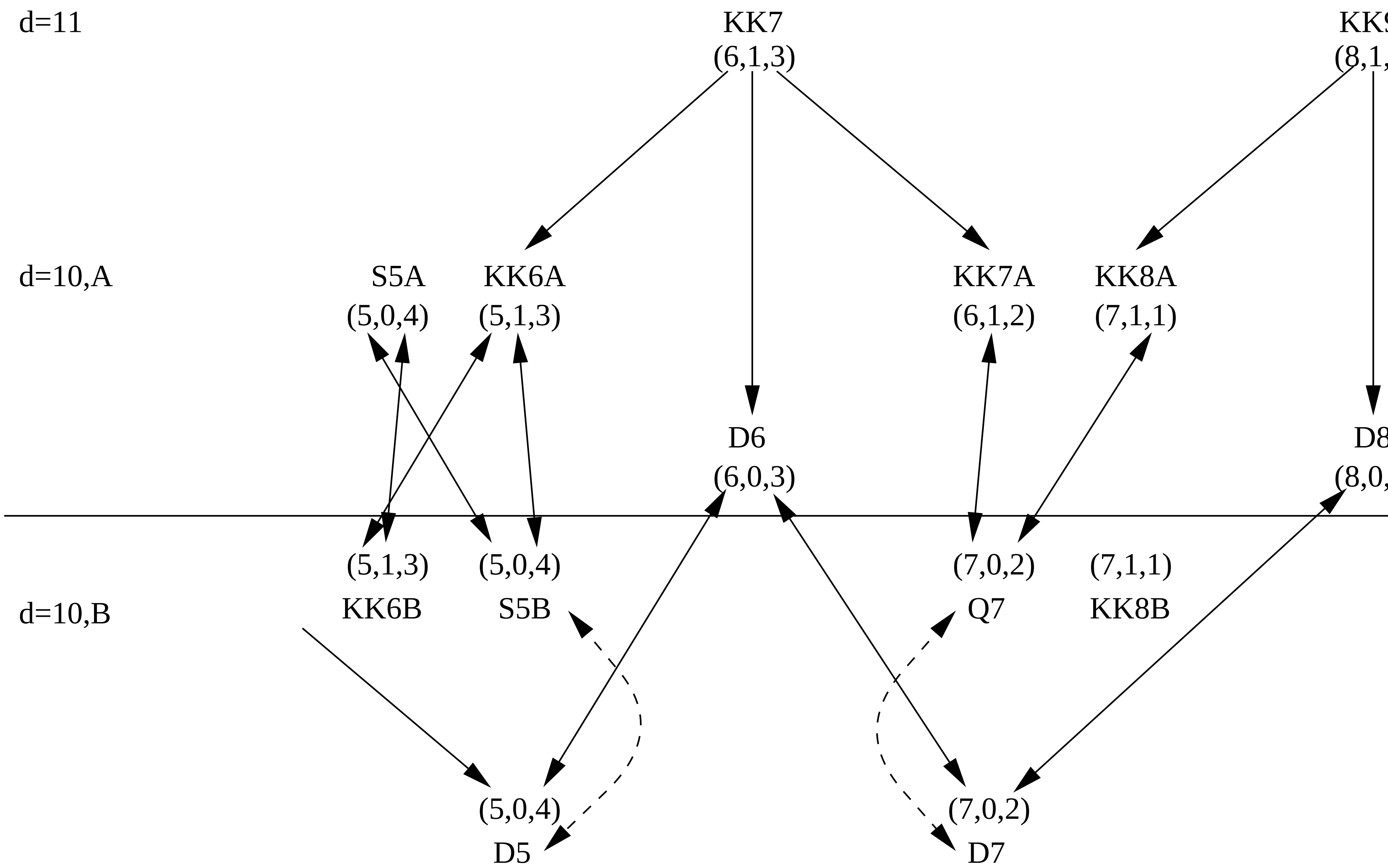
$$\int d^{11}\hat{\hat{x}}\sqrt{|\hat{\hat{g}}|}\,\Big\{\hat{\hat{R}}\left(\hat{\hat{\Omega}}\right) - \left(d\hat{\hat{k}}_{(n)}\right)_{\hat{\hat{\mu}}\hat{\hat{\nu}}} Q^{nm}\left(i_{\hat{\hat{k}}_{(m)}}\hat{\hat{C}}\right)^{\hat{\hat{\mu}}\hat{\hat{\nu}}} - \tfrac{1}{2\cdot 4!}\hat{\hat{G}}^2$$

$$-2\hat{\hat{K}}_{\hat{\hat{\mu}}\hat{\hat{\nu}}\hat{\hat{\rho}}}\hat{\hat{K}}^{\hat{\hat{\nu}}\hat{\hat{\rho}}\hat{\hat{\mu}}} + \tfrac{1}{2}\left(\hat{\hat{k}}_{(n)\ \hat{\hat{\mu}}}Q^{nm}\hat{\hat{k}}_{(m)}{}^{\hat{\hat{\mu}}}\right)^2 - \left(\hat{\hat{k}}_{(n)\ \hat{\hat{\mu}}}Q^{nm}\hat{\hat{k}}_{(m)\ \hat{\hat{\nu}}}\right)^2$$

$$+\tfrac{1}{6^4}\frac{\hat{\hat{\epsilon}}}{\sqrt{|\hat{\hat{g}}|}}\Big\{\partial\hat{\hat{C}}\partial\hat{\hat{C}}\hat{\hat{C}} + \tfrac{9}{8}\partial\hat{\hat{C}}\hat{\hat{C}}\left(i_{\hat{\hat{k}}_{(n)}}\hat{\hat{C}}\right)Q^{nm}\left(i_{\hat{\hat{k}}_{(m)}}\hat{\hat{C}}\right)$$

$$+\tfrac{27}{80}\hat{\hat{C}}\left[\left(i_{\hat{\hat{k}}_{(n)}}\hat{\hat{C}}\right)Q^{nm}\left(i_{\hat{\hat{k}}_{(m)}}\hat{\hat{C}}\right)\right]^2\Big\}\Big\}\,,$$

Having this theory we can find the T duality rules and check the following duality connections:



d=11
KK7
(6,1,3)
KK9
(8,1,
d=10,A
S5A
(5,0,4)
KK6A
(5,1,3)
KK7A
(6,1,2)
KK8A
(7,1,1)
D6
(6,0,3)
D8
(8,0,
(5,1,3)
KK6B
(5,0,4)
S5B
(7,0,2)
Q7
(7,1,1)
KK8B
d=10,B
(5,0,4)
D5
(7,0,2)
D7

Start with the 11-dimensional KK monopole which we call KK-7M-brane. This is a 7-dimensional, purely gravitational object, but one of the spacelike worldvolume directions, with coordinate z is compactified on a circle. Its metric is given by

$$
\begin{array}{c} KK7M \\ (6,1,3) \end{array}
\left\{
\begin{array}{rcl}
d\hat{\hat{s}}^2 & = & \eta_{ij}dy^i dy^j - H^{-1}\left(dz^2 + A_m dx^m\right)^2 \\
 & & -H d\vec{x}_3{}^2 \,, \\
2\partial_{[m}A_{n]} & = & \epsilon_{mnp}\partial_p H \,,
\end{array}
\right.
$$

We can reduce this solution in three different ways. In isometry direction, z, one gets the D-6-brane. Reducing on one of the standard spacelike worldvolume directions (double dimensional reduction) gives the KK-6A-brane, (the 10-dimensional KK monopole).

If we reduce it on a transverse coordinate, x^3:

$$
\begin{array}{c} KK7A \\ (6,1,2) \end{array}
\left\{
\begin{array}{rcl}
d\hat{s}^2_{IIA} & = & \left(\frac{H}{H^2+A^2}\right)^{-1/2}\left[\eta_{ij}dy^i dy^j - \frac{H}{H^2+A^2}dz^2 - H d\omega d\overline{\omega}\right] \,, \\
e^{\hat{\phi}} & = & \left(\frac{H}{H^2+A^2}\right)^{-3/4} \,, \\
\hat{C}^{(1)}{}_{\underline{z}} & = & \frac{A}{H^2+A^2} \,, \\
\partial_\omega A & = & i\partial_\omega H \,,
\end{array}
\right.
$$

where $\omega = x^1 + ix^2$ and $A = A_3$ and the last equation is simply $2\partial_{[m}A_{n]} = \epsilon_{mnp}\partial_p H$ with the assumption that H does not depend on x^3 and in the $A_1 = A_2 = 0$ gauge.

To relate it with type IIB solutions, we further reduce it in the isometry direction z. The resulting solution is a 9-dimensional "Q-6-brane":

$$Q6_9 \atop (6,0,2) \quad \left\{ \begin{array}{rcl} ds^2_{II} & = & \left(H^2+A^2\right)^{1/2}\left[H^{-1/2}\eta_{ij}dy^i dy^j - H^{1/2}d\omega d\overline{\omega}\right] \,, \\ e^{\phi} & = & \left(\frac{H}{H^2+A^2}\right)^{-1} \,, \\ C^{(0)} & = & \frac{A}{H^2+A^2} \,, \\ \partial_\omega A & = & i\partial_\omega H \,. \end{array} \right.$$

Notice that we have obtained two different solutions by reducing first on z and then on x^3 and in the inverse order. The difference is a rotation in internal space z, x^3 and, by T duality to an S duality transformation in the type IIB side.

We can now uplift this solution using the type IIB rules and adding the coordinate y. We obtain the Q-7-brane solution.

Conclusion

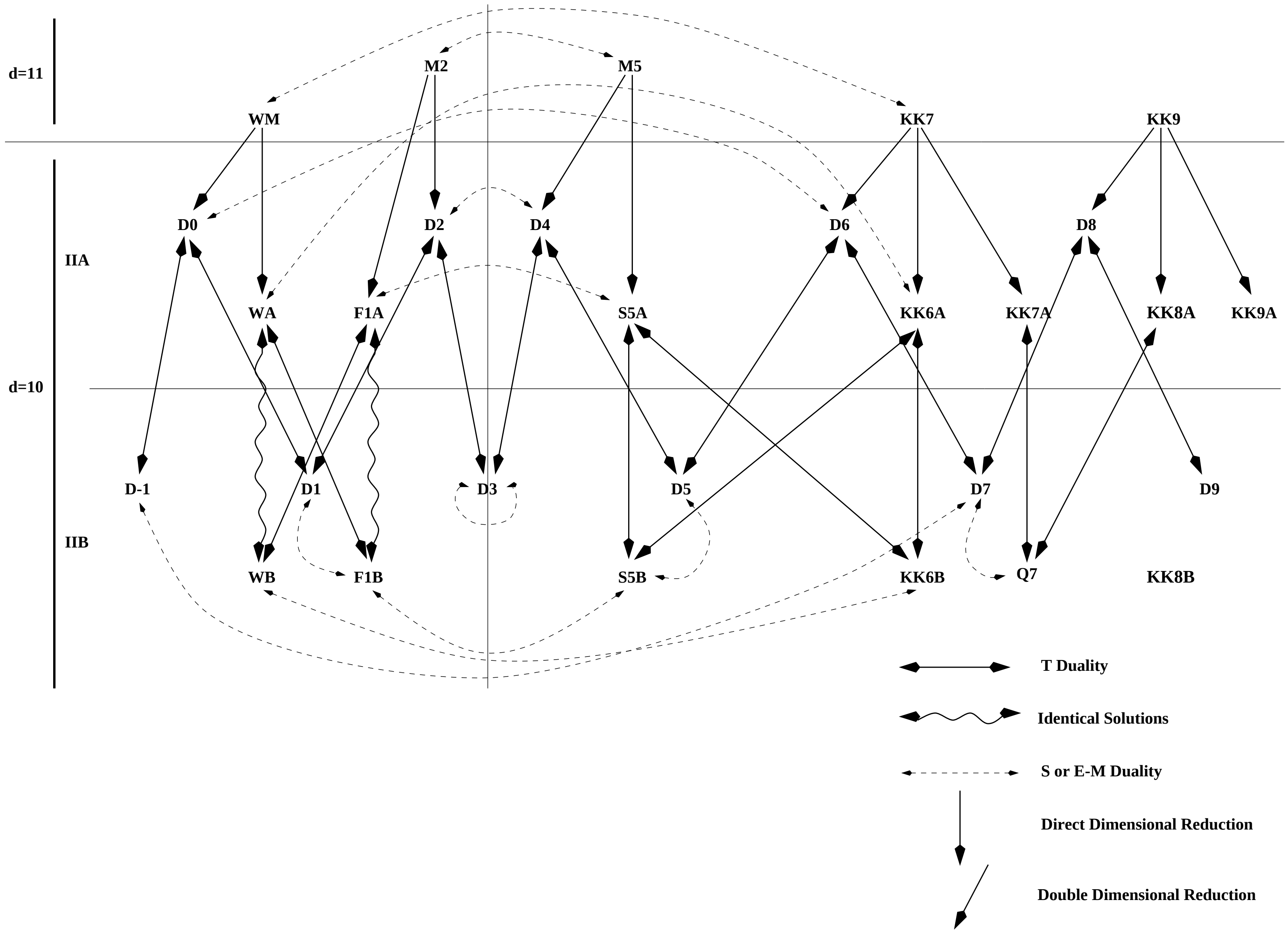
- We have explored GDR and applied it, in the most general possible way to $\hat{d} = 10, N = 2B$ SUGRA to obtain massive 9-dimensional SUGRAs. The different theories are related by $Sl(2,\mathbb{Z})$ transformations and labeled by the mass matrix which tells us which 7-branes we have in the background.

- We have studied the 7-brane solutions and found a new one, the "Q-7-brane" which is related to the KK monopole. D-7-branes and Q-7-branes enter into an $Sl(2,\mathbb{Z})$ triplet of charges. The third charge does not seem to be independent.

- We have found and interpreted the 11-dimensional origin of the massive 9-dimensional theory.

- General perspective on massive SUGRAs: U duality covariant massive $d = 4, N = 8$ SUGRA?

- Now we have a more complete picture of the extended solitons of M theory and string theory $\longrightarrow$


d=11
M2
M5
WM
KK7
KK9
D0
D2
D4
D6
D8
IIA
WA
F1A
S5A
KK6A
KK7A
KK8A
KK9A
d=10
D-1
D1
D3
D5
D7
D9
IIB
WB
F1B
S5B
KK6B
Q7
KK8B
T Duality
Identical Solutions
S or E-M Duality
Direct Dimensional Reduction
Double Dimensional Reduction